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Report No. T-19

ON THE PROBLEM OF COMET ORBIT DETERMINATION FOR SPACECRAFT INTERCEPT MISSIONS

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ON THE PROBLEM OF COMET ORBIT DETERMINATION FOR SPACECRAFT INTERCEPT MISSIONS

by

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SUMMARY

The scientific exploration of comets by means of spacecraft intercept mission presents problems in several important technical areas. One of the key problems in planning such a mission is the magnitude of the uncertainty or error in our present knowledge of the orbital motion of many periodic comets of interest. This uncertainty is a major determinant of how accurately a spacecraft may be guided to intercept a comet. In order to obtain the best viewing conditions of a comet's nucleus, the "miss distance" between the spacecraft and comet should be about 1000 km, and no greater than 10,000 km. This requirement is several orders of magnitude smaller than the errors associated with comet position ephemerides (prediction based on past observations).

This report discusses the factors which contribute to the inaccuracy of comet orbit determination and prediction, presenting illustrative numerical results for the two short period comets, Encke and D'Arrest. The main contributing factors are (1) the restricted arc of the total orbit over which a comet can be observed from Earth, (2) the relative inaccuracy in measuring right ascension and declination, possibly including

large systematic errors, (3) the sensitivity due to planetary perturbations, (4) the possibility of ill-defined non-gravitation forces or secular accelerations acting on the comet, and (5) computational errors of numerical integration. Generally, it is shown that miss distances under 10,000 km cannot be achieved unless the comets are observed during the year in which the spacecraft is launched.

The numerical analysis is facilitated by the COMET ORBIT DETERMINATION PROGRAM which has been developed for use on the IBM 7094 computer. The computer program is designed to integrate the orbit of a comet under the combined gravitational influence of the Sun and planets and other non-gravitational forces, and to process either actual or simulated comet observations in order to determine the most probable estimate of the comet's past or future motion. Also computed is a measure of the orbit determination uncertainty and the resultant miss distance for future missions of interest.

In the case of comets Encke and D'Arrest, the mission examples chosen are in 1974 and 1976, respectively. Past observations of Comet Encke are obtained and processed for seven appearances over the period 1931-1961, and for Comet D'Arrest, four appearances over the period 1910-1950. The best results of the data fitting process for each of these comets are obtained for the last several appearances in the above series.

The best estimate of Encke's orbit and its statistical uncertainty obtained for the data fit of the 1947, 1957, and

1961 apparitions is extrapolated to the 1974 apparition and mission of interest. Summary Table I lists the estimated values of miss distance due to the ephemeris error of Encke. With a priori information from the previous apparitions, the miss distance is as large as 70,000 km if no new observations are made in the year of launch. An observation schedule beginning at recovery of the comet and ending one week bwfore launch will reduce the miss to 7,000 km. Further observations beyond the launch data act to reduce the miss, slowly at first, and then rapidly as the observation geometry improves with the decreasing distance between Earth and Encke. To achieve a desirable miss distance of under 1,000 km, the observation schedule must extend to the later portion of the flight--within 20 days of encounter. This implies a late midcourse correction, but only about 4 m/sec.

For comparison purposes, Summary Table I also includes the estimated miss distance when no apriori information is assumed. This would correspond to a worse-case situation wherein no confidence is given to a previous orbit determination. Although it is unlikely that a mission would ever be planned under such adverse conditions (recovery of the comet could not be assured), the results are useful in placing an upper bound on the orbit determination problem.

Summary Table II presents similar results for a 1976 mission to Comet D'Arrest. The initial miss distance estimate of 108,000 km is based on the observational data fit of the

1943 and 1950 appearances. However, this result may be too optimistic since the 1943-50 orbit could not be accurately linked with the observations taken in earlier appearances. In any event, the analysis shows that observation of D'Arrest taken in the year of launch would be very effective in reducing the miss distance uncertainty. Even in the worst case of no a priori information (assuming that the comet can be recovered), a 1000 km miss distance is still attainable but requires a ΔV correction of about 40 m/sec made 14 days before encounter.

This report recommends that further attention should be given to the orbit determination of each of these comets, especially Comet D'Arrest. The present analysis can be updated with later observational data which were not available at the time this analysis was performed. In addition, an effort should be made to improve the ephemerides of other comets which are of interest for future exploratory space missions. This applies particularly to comets which do not have excellent observational geometries in the year of launch as do comets Encke and D'Arrest. Such an effort will result in the increased probability of recovering the comet during the year of spacecraft launch and tend to reduce the ΔV requirement of late trajectory corrections.

Summary Table I

ESTIMATED MISS DISTANCE FOR 1974
MISSION TO COMET ENCKE

- Launch Date T_L, 1974 Feb. 7
- Encounter, $T_L + 110^d$
- Recovery, $T_L - 160^d$
- Observations at 8 Day Intervals Beginning at Recovery
- Observation Error, 2 Sec Arc
- A Priori Data, Orbit Determination from Observations in 1947, 1957, 1961 Appearances

Number of Observations	Miss Distance (1σ)	
	A Priori Data	No A Priori
None in Year of Launch	70,000 km	∞
19, Ending $T_L - 8^d$	7,000	17,000
21, Ending $T_L + 8^d$	6,500	14,000
28, Ending $T_L + 64^d$	3,200	10,000
31, Ending $T_L + 90^d$	1,000	4,000
32, Ending $T_L + 98^d$	500	2,000

Summary Table II

ESTIMATED MISS DISTANCE FOR 1976 MISSION TO COMET D'ARREST

- Launch Date T_L, 1976 April 21
- Encounter, $T_L + 130^d$
- Recovery, $T_L - 100^d$
- Observations at 8 Day Intervals Beginning at Recovery
- Observation Error, 2 Sec Arc
- A Priori Data, Orbit Determination from Observations in 1943, 1950 Appearances

Number of Observations	Miss Distance (1σ)	
	A Priori Data	No A Priori
None in Year of Launch	108,000 km	∞
11, Ending $T_L - 12^d$	4,500	125,000
14, Ending $T_L + 12^d$	3,300	46,000
18, Ending $T_L + 44^d$	1,900	14,000
22, Ending $T_L + 76^d$	1,000	4,300
27, Ending $T_L + 116^d$	480	1,000

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1. INTRODUCTION

This report is one in a series of reports by the Astro Sciences Center of IIT Research Institute on a study of periodic comets and their scientific exploration by means of spacecraft intercept missions (cf. Appendix B). Earlier reports in this series have presented the scientific objectives of missions to the comets, a compendium of existing cometary data, trajectory and sighting analysis, and a survey of suitable comet missions including experimental payload selection and questions of mission constraints. The overall objectives of the cometary studies have been to show the best way in which spacecraft intercept missions can complement and significantly add to the present understanding of comets, to outline the mission profiles for those intercept missions which are considered worth while, and to investigate and recommend solutions to key problem areas which have bearing on mission success.

The present report is addressed to one such problem area, namely, the uncertainty in knowledge of a comet's orbital position ephemeris. This uncertainty, often quite large, has a major influence on how accurate a spacecraft can be guided on its intercept trajectory. In order to achieve good viewing of the comet nucleus, a small miss distance on the order of 1000 km and certainly no greater than 10,000 km, would be desirable.

It has always been difficult to predict accurately the time and place of the return of a periodic comet. The differences between observation and theory are sometimes as large as several days in the time of perihelion passage. For many comets this would translate into an ephemeris error of several million kilometers. In contrast to this situation, the ephemeris errors of the major planets are never more than several hundred kilometers.

The problem of accurate determination of cometary motion is made especially difficult by the fact that most comets are observed only in the vicinity of perihelion and, at that, over a very short arc of its total orbit. A consequence of this observation restriction is that the orbital elements cannot usually be determined accurately during one apparition or appearance of the comet. The element which suffers the greatest amount of indeterminacy is the semi-major axis or, equivalently, the mean angular motion. As a result, the prediction of future position-in-orbit is rather poor. This problem can be alleviated by linking observations obtained over several appearances.

Additional causes for the discrepancy between observation and theory are the following:

(1) there may be non-gravitational forces acting to perturb the orbit, these forces being unique to the nature of comets themselves. For example, mass loss under the influence of solar radiation,

or drag from a resisting medium in the near vicinity of the Sun.

(2) the planetary perturbations acting on the comet may be computed only approximately. Of particular importance here is the planet Jupiter which strongly influences an entire family of comets. For example, the mass of Jupiter may not be known with sufficient accuracy. Also, and probably more important, any error in the estimate of the comet's orbit will be magnified after a close approach to Jupiter.

(3) there may be fairly large systematic errors in the observations themselves. This could be caused by the diffuse image presented by the comet and also the likelihood that the center of mass does not coincide with the center of light. Also, the catalog positions of background stars to which the comet image is referred may contain systematic errors.

(4) computation errors due to roundoff and truncation in the numerical integration scheme.

In order to facilitate a numerical study of cometary motion, a moderately high precision Orbit Determination Program has been developed for use on the IBM 7094 computer. The term "orbit determination" as used in this context implies first, a definitive or most probable orbit rather than a preliminary orbit and, second, a linear differential correction to an initial orbit estimate. The orbit correction procedure is based on the theory of optimal linear filtering and prediction sometimes referred to as "sequential, minimum-variance estimation". The Orbit Determination Program was designed to be used for the following purposes:

(1) to compute an orbit by numerical integration for a comet under the combined gravitational influence of the Sun and planets, and other non-gravitational forces.

(2) to process actual comet observations in order to determine the most probable estimate of the comet's motion either in the past or future.

(3) to process simulated comet observations to be made prior to and following the launch of a spacecraft in order to determine the miss distance due to the comet's position uncertainty.

Appendix A of this report describes the Orbit Determination Program in a fair amount of detail, and is recommended to the reader for a better understanding of the method of analysis employed in this study.

The first phase of the present investigation is addressed to the problem of comet orbit determination given a set of actual observations taken over a period of several past appearances. In particular, we consider the two well known short-period comets, Comet Encke and Comet D'Arrest, which have been observed numerous times over the past one hundred years or more. These comets are studied with a view towards obtaining the most representative orbit and its probable uncertainty. The second phase of the investigation then uses this information to predict the future motion of the comet and, specifically, to estimate the comet's ephemeris errors which are relevant to the guidance accuracy and fuel requirements of a spacecraft intercept mission. A similar type of analysis, but not including actual past observations, has been performed by the Jet Propulsion Laboratory for possible missions to four other comets (Light 1966). Our results and conclusions are in general agreement.

2. ORBIT DETERMINATION FROM PAST OBSERVATIONS

2.1 Comet Encke

2.1.1 Previous Investigations

Of all the short-period comets, Comet Encke, having the shortest period of 3.3 years, has received the most attention in terms of its observation and orbit analysis. This comet has been observed at 46 appearances since its discovery in 1786. Previous studies of Comet Encke have shown, with a fair degree of certainty, that its motion cannot be adequately represented by gravitational theory alone (Recht 1939). That is to say, there has been a noted discrepancy between the observed and predicted positions which usually grows with time. Attempts to explain the phenomena by systematic errors in both observations and computation have met with little success. It is possible, of course, that there was a lack of sufficient knowledge to make such a systematic error analysis. In any event, it has been necessary to assume a secular acceleration of mean motion, and often of eccentricity, in order to represent accurately the orbit by observations.

In the case of Comet Encke, this acceleration is of positive sense to account for the fact that the comet has apparently moved faster than predicted. The acceleration model that has been assumed in past investigations is of the form

$$n = n_o + n' \left(\frac{t}{1200}\right)$$

where n is the mean daily motion and 1200 days is very nearly the period of the comet. The value determined for n' over several sets of successive appearances has varied considerably indicating that the acceleration is not uniform. Thus, over the period 1819-58, the value of n' was found to be nearly constant at 0.1"/day per orbit. Over the period 1924-34, n' has been given a value of 0.038. Equivalently, the change in period per orbit would range from -0.1 to -0.04 day. While this change seems small, it would result in significant position errors after not too many orbits.

Previously determined orbital elements of Comet Encke are listed in Table 1 for seven apparitions over the period 1931-61 (Porter 1961). These elements are used in the present study to initialize the Orbit Determination Program, and also serve for comparison purposes. While the reference source for these elements does not give any quantitiative measure of their accuracy, it does give a qualitative designation. Thus, the designation A5 which applies to the five apparitions (1931, 34, 37, 41 47) indicates a detailed orbit analysis based on many observations with linking of at least two apparitions by a perturbation scheme. There is no indication, however, as to whether a secular acceleration model was assumed for the determination of these elements.

2.1.2 <u>Observations Used in Present Analysis</u>

For the purpose of the present study, about 100 published observations of Comet Encke during the period 1931-61

Table 1

PORTER'S ORBITAL ELEMENTS FOR COMET ENCKE

EPOCH	T	a, (AU)	e	Ω, deg	i, deg	ω, deg
1931 (A.5) (assumed at perihelion)	June 3.11363	2.209292	0.849786	334.8974	12.5633	184.9133
1934 (A.5)	Sept. 15.2816	2.209563	0.849806	334.8823	12.5599	184.9459
1937 (A.5)	Dec. 27.7538	2.210105	0.849604	334.8810	12.5488	184.9393
1941 (A.5)	Apr. 17.1426	2.219187	0.846182	334.7674	12.3511	185.1577
1947 (A.5)	Nov. 26.32673	2.218639	0.846295	334.7432	12.3513	185.1902
1957 (p)	Oct. 19.8454	2.215732	0.847399	334.7290	12.3748	185.2276
1961 (p)	Feb. 5.583	2.216612	0.847056	334.7214	12.3597	185.2271

(A.5) Large number of observations, planetary perburbations applied, linking of more than one apparition, least-squares fit probably used.

(p) Predicted elements, planetary perturbations applied, in some cases correction to T and a by means of one or two observations.

were collected (Astronomical Journal, Lick Observatory Bulletin). It was necessary first to check this data for obvious errors; e.g., due to misprints. This was done by computing the residuals (observed-calculated angles) in each apparition using Porter's elements for the reference orbit. Those observations showing large inconsistency with the general trend of the residuals were disregarded. Also, when observations were closely grouped and consistent, only one observation was taken to represent this group. Table 2 lists the set of observations finally chosen for the orbit determination. There are a total of 26 observations obtained in 7 appearances over the period 1931-61 with no less than 3 observations in each appearance. Observations are separated by at least one week and usually 2-6 weeks.

2.1.3 Secular Acceleration

It might be said at the start that little success was obtained in linking more than two apparitions of Comet Encke in the case when motion was assumed to be influenced only by gravitational forces (the Sun and perturbing planets). The linking of two apparitions is not too difficult since it is usually possible to adjust the mean motion at the expense of other orbital elements in such a way as to represent the observations with fair accuracy. It should be noted, however, that even in linking two apparitions there is evidence of a forced fit in that the final residuals show systematic runs rather

Table 2

OBSERVATIONS OF COMET ENCKE USED IN PRESENT ANALYSIS

Date (E.T.)	Astrometric Right Ascension and Declination Mean Equator & Equinox of 1950.0 α	δ	Observatory (City)
1931 June 21.93312	114°.09936	8°.32438	Cordoba
June 29.94836	125.75292	-2.83627	Cordoba
July 15.98721	173.21260	-36.40107	Cordoba
1934 July 8.34798	55.53954	27.31937	Williams Bay
July 18.34914	65.38815	29.43517	Williams Bay
Aug. 21.40329	118.57804	27.83642	Williams Bay
1937 Sept. 4.47974	35.00279	27.46838	Mount Hamilton
Oct. 11.23679	24.91648	38.26244	Williams Bay
Oct. 28.20667	358.34742	43.74835	Mount Hamilton
Nov. 8.09227	325.60922	39.60246	Charlottesville
Nov. 23.97627	283.18687	14.86253	Charlottesville
1941 Jan. 20.03376	350.86642	4.20893	Williams Bay
Feb. 19.04412	2.46344	8.65115	Williams Bay
Mar. 1.05537	7.59581	10.63437	Williams Bay
1947 Aug. 14.40406	46.50589	28.00142	Mount Hamilton
Sept. 21.42720	74.24691	41.09196	Mount Hamilton
Oct. 9.37491	114.85613	47.61468	Williams Bay
Oct. 23.39685	166.24446	32.99894	Williams Bay
Nov. 15.53985	207.51777	-5.14698	Flagstaff (Lowell)
1957 July 28.40112	54.92128	28.48136	Washington (USNO)
Aug. 31.45050	94.76369	34.80072	Washington (USNO)
Sept. 19.43099	133.81106	28.40439	Washington (USNO)
1960 Oct. 22.15150	357.65705	17.61303	Williams Bay
Nov. 21.05646	341.00456	9.91919	Williams Bay
Dec. 8.99746	336.38719	6.32133	Williams Bay
1961 Jan. 4.00804	333.87148	2.64206	Williams Bay

than the expected random distribution. The real test comes in the attempt to link more than two apparitions.

Since the principal effect of the observed discrepancy in the motion is an advance or regression of the comet's position-in-orbit, a tangential secular acceleration of the following form is assumed

$$\underline{A} = (p_o + p_i r^{-1} + p_2 r^{-2} + . . .)\ \underline{V} \qquad (1)$$

Here, the $\{p_i\}$ are constant coefficients to be determined by data fitting, r is the heliocentric radius of the comet and $\underline{V}$ is its velocity vector. A maximum of 10 coefficients is allowed in the Orbit Determination Program; however, only the p_o term was used in the analysis. To express the secular acceleration parameter in more familiar terms, namely the change in period per orbit ΔP, and to allow comparison with previous results, the following relationships can be derived

$$p_o = \Delta P/3P^2 = -n'/6\pi \qquad (2)$$

where n' is the change in mean motion per orbit in units of radians/day.

Table 3 illustrates the evidence of a secular acceleration effect for Comet Encke. In this example, the observations of the 1931, 1934 and 1941 apparitions were used to determine the orbit over this period both with and without a secular acceleration. The two orbits so obtained were then extrapolated to the 1947 apparition and in each case the 1947 residuals were computed. When a secular acceleration was allowed, the

Table 3

EVIDENCE OF SECULAR ACCELERATION OF COMET ENCKE

° COMPARISON OF 1947 RESIDUALS FOR PREDICTED ORBIT CARRIED FORWARD FROM DATA FIT OVER 1931, 1934, 1941 APPARITIONS WITH AND WITHOUT SECULAR ACCELERATION.

° SECULAR ACCELERATION, $\Delta P = -0.025$ DAYS/ORBIT

DATE	$\Delta\alpha$		$\Delta\delta$	
	NO SECULAR ACCELERATION	WITH SECULAR ACCELERATION	NO SECULAR ACCELERATION	WITH SECULAR ACCELERATION
1947 Aug. 14	505"	-15"	131"	-10"
Sept. 21	1700	-72	99	- 3
Oct. 9	3142	-147	-907	54
Oct. 23	1214	-45	-2055	118
Nov. 15	109	+ 5	-809	51
RMS AVERAGE	1900"	76"	1380"	63"

value obtained for the 1931-41 peroid was -0.025 day/orbit. In this case, the 1947 residuals have an average RMS value of about 70". This is to be compared with an average residual of about 1650" for the case in which no secular acceleration was allowed. Hence, the 20-fold improvement in representing the 1947 observations gives ample evidence of the existence of a non-gravitational force acting to perturb the motion of Comet Encke, and also that this force has a significant tangential component. Of course, no claim can be made that the secular acceleration model assumed here actually represents the real forces that are acting. What is hoped for is that the secular acceleration be fairly uniform so that its average effect can be estimated.

Several values of the secular acceleration determined by fitting the observational data over different combinations of appearances are listed in Table 4. The values so obtained appear to be consistent although the period 1947-61 gives a value about 30 percent lower than the period 1931-47. The mean value of the five combinations of appearances is -0.0223 day/orbit or, in terms of the mean motion, +0.02"/day per orbit. Thus, there is fairly good agreement with previous investigations.

2.1.4 Systems of Appearances

The procedure followed in the remaining analysis of Comet Encke was to separate the observations into two main systems of appearances. Keeping chronological order, the period 1931-47 is designated System I, and that for 1947-61, System II. The observations in each of these two systems were processed by

Table 4

DETERMINED VALUES OF SECULAR ACCELERATION FOR COMET ENCKE

SYSTEM OF APPARITIONS	SECULAR ACCELERATION p_o (day^{-1})	SECULAR ACCELERATION ΔP ($\frac{days}{orbit}$)
1931, 1934, 1941	-5.73×10^{-9}	-0.0248
1931, 1934, 1941, 1947	-5.83×10^{-9}	-0.0251
1931, 1934, 1937 1941, 1947	-5.63×10^{-9}	-0.0243
1947, 1957, 1961	-4.38×10^{-9}	-0.0189
1957, 1960 START FROM 1954 PORTER'S ELEMENTS	-4.22×10^{-9}	-0.0182
MEAN VALUE	-5.16×10^{-9}	-0.0223
STANDARD DEVIATION	0.7×10^{-9}	0.003

the "sequential minimum-variance estimation" method to obtain the most representative orbit over the respective periods (see Appendix A). The initial orbit estimates in 1931 and 1947 are taken from Porter's elements listed in Table 1. Each observation is given equal weight corresponding to a 1σ random observation error of 2". The values of the secular acceleration obtained for the two systems of appearances has already been given in Table 4.

To check the "goodness-of-fit", the orbit determined at the end of each system of appearances was integrated backward and the O-C residuals in right ascension and declination were computed. The residuals by final elements for System I are listed in Table 5. Considering the System I orbit determination, it is clear that the residuals are not as small as one would hope for. The largest residuals of about 200" occurs in 1931, but the RMS average over the entire period is considerably smaller. Inasmuch as the System I orbit is being extrapolated backward from 1947, one would expect the largest deviation to occur in 1931. Actually, the residuals by themselves do not give a complete picture of the orbit determination accuracy. This is because the sensitivity of α and δ to errors in the elements are not constant throughout the period. Rather, they show considerable variation with the geometric conditions existing at the moments of observation. To illustrate this situation, the sensitivity α and δ to a +0.01 day error in perihelion time is shown in Table 6 for one date in each year

Table 5

RESIDUALS OF SYSTEM I - COMET ENCKE

• Secular Acceleration $\Delta P = -0.0243$ day/orbit

DATE		RESIDUALS BY FINAL ELEMENTS	
		$\Delta\alpha$	$\Delta\delta$
1931	June 21	21"6	88"6
	June 29	21.0	124.7
	July 15	-176.3	215.8
1934	July 8	-28.4	0.9
	July 18	-27.6	-2.3
	Aug. 21	-63.2	27.9
1937	Sept. 4	-4.6	-6.8
	Oct. 11	-28.1	-19.4
	Oct. 28	-10.6	-37.9
	Nov. 8	54.5	-52.3
	Nov. 23	103.9	11.2
1941	Jan. 20	0.8	3.0
	Feb. 19	-2.2	2.4
	Mar. 1	-14.3	-0.7
1947	Aug. 14	20.0	-1.7
	Sept. 21	26.7	-2.6
	Oct. 9	6.4	-4.1
	Oct. 23	-7.9	13.9
	Nov. 15	3.6	14.2

of appearance. Thus, for example, one finds that the smallest sensitivity occurs in 1941. This information, taken together with similar sensitivities to the other orbital elements, show that the geometric conditions of the 1941 apparition was not very conducive to accurate orbit determination. Thus, the apparently small 1941 residuals shown in Table 5 does not imply that the 1941 apparition is better determined than say the 1937 apparition where the residuals are fairly large.

To further expound upon this point, let us compare the various perihelion times of System I with those of Porter (Table I) which are presumably very accurate.

Apparition	$(T_{System\ I} - T_{Porter})$
1931	-0.021 day
1934	-0.016
1937	-0.007
1941	-0.006
1947	-0.003

The above table shows that the System I orbit becomes less accurate with each earlier apparition, however, the largest error is only 0.02 day. It must be remembered that the final orbit of System I assumes a constant secular acceleration determined as an average over the entire period. With this consideration, it could be said that the System I orbit is a fairly accurate representation of the 1931-47 apparitions. It might be possible, of course, to find a better system of elements either by continued iteration or by employing a better secular

Table 6

SENSITIVITY OF RIGHT ASCENSION AND DECLINATION TO ERROR IN TIME OF PERIHELION - COMET ENCKE

$\Delta T = +0.01$ DAYS

DATE	$\Delta\alpha$	$\Delta\delta$
1931 July 15	-86"	98"
1934 Aug. 21	-45	15
1937 Nov. 8	114	-57
1941 Mar. 1	-3	-2
1947 Oct. 23	-42	73
1957 Sept. 19	-49	24
1961 Jan. 4	32	8

acceleration model. Numerical integration error is also a factor to be considered.

The System II residuals given in Table 7 show that the 1957 and 1961 apparitions are very well determined. Here, the largest residual is 5.4" and the average RMS residual over these two apparitions is only 3". The System II orbit projected back to 1947 does show fairly large residuals by comparison. However, following along the lines of the previous discussion, it is found that the 1947 perihelion time is in error by only 0.01 day.

A comparison of the System I and II orbital elements for the 1947 apparition is shown in Table 8. The elements T, a, and e differ by the amounts 0.013 day, -3×10^{-5} and -7×10^{-6}, respectively. The orientation angles Ω and ω show differences of about 50".

Finally, Table 9 shows the 1961 orbital elements of Comet Encke determined by the System II data fit. The 1σ uncertainty in the elements are also given. These elements and uncertainties may be used to predict the future motion of Comet Encke. For example, suppose we extrapolate the orbit to the 1974 apparition. The uncertainty in the semi-major axis and the secular acceleration will each result in a perihelion time error of about 0.01 days. For Comet Encke, this time error corresponds to a 60,000 km position-in-orbit error at the 1974 perihelion.

Table 7

RESIDUALS OF SYSTEM II - COMET ENCKE

° SECULAR ACCELERATION $\Delta P = -0.0189$ DAY/ORBIT

DATE		Residuals by Final Elements	
		$\Delta\alpha$	$\Delta\delta$
1947	Aug. 14	29.9"	5.3"
	Sept. 21	86.5	4.2
	Oct. 9	139.0	-40.1
	Oct. 23	53.5	-83.8
	Nov. 15	9.8	-31.9
1957	July 28	2.3	-1.1
	Aug. 31	-4.7	0.0
	Sept. 19	-3.6	3.5
1960	Oct. 22	-0.6	-2.4
	Nov. 21	-4.1	-2.6
	Dec. 8	-0.1	-3.4
1961	Jan. 4	1.3	-5.4

Table 8

COMPARISON OF SYSTEM I, II ORBITAL ELEMENTS
AT 1947 APPARITION OF COMET ENCKE

ELEMENTS	SYSTEM I 1931-47	SYSTEM II 1947-61	DIFFERENCE (II-I)
EPOCH	1947 Nov. 15.0	1947 Nov. 15.0	
T (ET)	1947 Nov. 26.32341	1947 Nov. 26.33671	+0.0133
a (AU)	2.2185605	2.2185309	-0.0000296
e	0.8463087	0.8463018	-0.0000069
Ω	334°73449	334°75073	+58"
i	12°35188	12°35399	+7"6
ω	185°19835	185°18476	-49"
P (days)	1206.9946	1206.9705	-0.0241
q (AU)	0.3409735	0.3409842	+0.0000107

Table 9

<u>1961 ORBITAL ELEMENTS OF COMET ENCKE</u>
<u>SYSTEM II DATA FIT</u>

ORBITAL ELEMENTS		UNCERTAINTY (1σ)
EPOCH	1961 Feb. 10.0	
T (ET)	1961 Feb. 5.59496	7.5×10^{-4} day
a (AU)	2.2165752	3×10^{-6}
e	0.8470703	3×10^{-6}
Ω	$334\overset{\circ}{.}73141$	$7\overset{''}{.}5$
i	$12\overset{\circ}{.}36261$	$1\overset{''}{.}1$
ω	$185\overset{\circ}{.}22031$	$8\overset{''}{.}0$
	SECULAR ACCELERATION	
P_o	-4.38×10^{-9} day^{-1}	0.6×10^{-9}
ΔP	-0.0189 day/orbit	0.0025

2.2 Comet D'Arrest

2.2.1 Previous Investigations

Comet D'Arrest, having a period of about 6.6 years, has been observed at 10 appearances since its discovery in 1851. Occasionally the comet passes within 0.5 AU of the planet Jupiter which strongly perturbs its subsequent motion. A close approach was made in 1861 and 1920, and is expected again in 1968. In fact, it is the perturbing effect of the 1968 approach which offers an improved Earth-comet geometry for a 1976 space probe mission.

The most extensive study of the motion of Comet D'Arrest was made by Recht, who attempted to find a single system of osculating elements that would represent closely the observations obtained in the eight appearances over the period 1851-1924 (Recht 1939). Although this attempt was not successful, his careful analysis of the motion for pairs of successive appearances indicated the presence of a secular acceleration of mean motion and also of eccentricity. Interestingly enough, the apparent secular accelerations were of the same magnitude as for Comet Encke, but opposite in direction. The average secular change of the mean motion over the 73 year interval studied was determined to be -0.035"/day per orbit (or, equivalently, ΔP = 0.15 day/orbit). In his analysis, Recht did not attempt to find out whether the application of this uniform change would make it possible to represent closely the observations in each appearance of the comet. It is unlikely that the residuals, if obtained, would have been uniformly small since the secular accelerations determined for the various pairs of appearances showed a significant variance from the average value. Further evidence that the orbit was not well determined is given by the fact that the predicted time of perihelion for the 1943 apparition was too large by 1.2 days (Porter 1961).

Table 10 gives Porter's elements of Comet D'Arrest for the four appearances in 1910, 1923, 1943, and 1950. These elements are used in the present study to initialize the Orbit Determination Program, and also serve for comparison purposes. It might be mentioned that the original reference for the 1943 and 1950 elements was an analysis by Recht.

2.2.2 Present Analysis

Published observations of Comet D'Arrest for the appearances of 1910, 1923, 1943 and 1950 were obtained from the Astronomical Journal and the Lick Observatory Bulletin. The five observations in each appearance used in the present analysis are listed in Table 11. It is noted that all but one of the observations were taken at the Yerkes Observatory in Williams Bay, Wisconsin.

The first step taken in the analysis was to compare the residuals of the 1923-50 appearances by a forward extrapolation of the supposedly well determined orbital elements of 1910. These elements are:

Table 10

PORTER'S ORBITAL ELEMENTS FOR COMET D'ARREST

EPOCH	T	a, (AU)	e	Ω, deg	i, deg	ω, deg
1910 (p) (assumed at perihelion)	Sept. 16.9060	3.497858	0.636915	146.9218	15.7865	173.7980
1923 (p)	Sept. 15.57	3.5304	0.6169	143.8823	18.0602	174.1262
1943 (p)	Sept. 22.574	3.559048	0.610627	143.6293	18.0114	174.4003
1950 (p)	June 6.395	3.553733	0.612275	143.6137	18.0545	174.4318

(p) Predicted elements, planetary perturbations applied, in some cases correction to T and a by means of one or two observations.

Table 11

OBSERVATIONS OF COMET D'ARREST
USED IN PRESENT ANALYSIS

Date (E.T.)			Astrometric Right Ascension and Declination Mean Equator & Equinox of 1950.0 α	δ	Observatory (City)
1910	Sept.	26.07145	277°76087	-24.16252	Williams Bay
	Oct.	5.05419	287.31374	-27.03062	"
	Oct.	31.00743	316.03135	-29.38802	"
	Nov.	7.01778	323.24870	-28.64804	"
	Nov.	25.00812	339.73392	-25.03925	"
1924	Jan.	2.01983	0.98843	-17.17726	"
	Jan.	4.05046	2.18625	-16.63166	"
	Jan.	6.00447	3.33072	-16.10641	"
	Jan.	24.01671	13.32222	-11.34379	"
	Jan.	27.03660	14.91440	-10.57001	"
1943	Oct.	24.10051	300.31108	-27.95931	Fort Davis
	Nov.	14.01066	319.56084	-27.64378	Williams Bay
	Nov.	28.00823	331.41064	-25.68282	"
	Dec.	25.99760	351.91439	-19.45389	"
1944	Jan.	15.02724	4.38148	-14.29949	"
1950	June	11.34734	5.97871	3.21817	"
	June	25.34247	17.08396	3.86342	"
	July	14.35196	30.87074	3.57436	"
	Aug.	16.36716	49.21903	-0.21375	"
	Sept.	7.36217	56.07282	-4.65408	"

EPOCH	1910 Sept. 16
T	1910 Sept. 16.89704
a	3.497793 AU
e	0.636944
Ω	$146^{\circ}.92180$
i	$15^{\circ}.78650$
ω	$173^{\circ}.79800$

The computed residuals listed in Table 12 show typical values of several hundred seconds of arc. Interestingly enough, the residuals do not show a uniform growth with time, which may be partially explained by the fact that the computed errors in perihelion time are not very uniform. For example, in 1923 and 1943, the respective values of ΔT are about 0.14 and 0.19 days. However, in 1950, ΔT is only 0.002 day. A further point to be made is that the α residuals are positive which would indicate the possibility of a positive secular acceleration of mean motion. This result would then be in contradiction to that found by Recht.

All attempts to link the four appearances by a single system of osculating elements were largely unsuccessful. This was true also when a secular acceleration was allowed. Variously determined values of the coefficient p_o, obtained by fitting over different combinations of appearances, differed by an order-of-magnitude and also of sign. Furthermore, the corresponding orbits did not represent the observations very well. Often, the residuals were larger than those obtained by simply extrapolating the 1910 elements. The principal conclusions of this work was that we could neither verify nor dispute the analysis by Recht, and that the long-term behavior of Comet D'Arrest will require further, and more careful study.

A tentative, but likely, identification of the source of the problem is due to the close approach of the comet to Jupiter in 1920. In the first place, and probably most significant, is the sensitivity problem. That is, a small error in the orbit estimate will be magnified by a close approach to Jupiter so that the subsequent prediction of the motion will be degraded. This problem might be alleviated by allowing several iterations in the orbit determination process, thereby improving the validity of the linearity assumptions. This was not attempted in the present analysis. In addition to the sensitivity problem, other sources of error might be identified with numerical integration error buildup over many orbits, and an insufficiently accurate position ephemeris for Jupiter.

In the face of the relative failure to link the four appearances of Comet D'Arrest, an attempt was made to link only the last two appearances in 1943 and 1950. This attempt was successful as is evidenced by the residuals listed in Table 13. The largest residual is only 4" and the RMS average is 2". No secular acceleration is included in this orbit determination. A comparison of the 1943 elements found in this analysis to Porter's elements show fairly large differences, e.g., 6.5×10^{-4} AU in semi-major axis, 1.5×10^{-4} in eccentricity, and 0.018 deg in argument of perihelion.

Table 12

RESIDUALS OF 1910 ORBIT - COMET D'ARREST

• No Secular Acceleration

DATE	$\Delta\alpha$	$\Delta\delta$
1910 Sept. 26	$-4\overset{''}{.}3$	$-3\overset{''}{.}4$
Oct. 5	2.0	-2.0
Oct. 31	2.1	-3.6
Nov. 7	0.5	-2.4
Nov. 25	1.9	-0.7
1924 Jan. 2	387	146
Jan. 4	366	143
Jan. 6	365	143
Jan. 24	310	146
Jan. 27	301	147
1943 Oct. 24	479	-95
Nov. 14	540	-8
Nov. 28	531	56
Dec. 25	459	127
1944 Jan. 15	395	148
1950 June 11	72	111
June 25	116	126
July 14	165	148
Aug. 16	209	186
Sept. 7	226	215

Table 13

COMET D'ARREST ORBIT DETERMINATION FROM OBSERVATIONS IN 1943, 1950

a) RESIDUALS BY FINAL ELEMENTS

DATE		$\Delta\alpha$	$\Delta\delta$
		$''$	$''$
1943	Oct. 24	+0.18	-2.00
	Nov. 14	-1.63	+0.04
	Nov. 28	-3.01	+0.30
	Dec. 25	-2.53	+2.87
1944	Jan. 15	-4.38	+1.04
1950	June 11	-1.10	+2.73
	June 25	-1.56	+0.52
	July 14	+2.15	+0.74
	Aug. 16	+1.78	-1.12
	Sept. 7	+0.29	-1.62
RMS AVERAGE		2.20	1.60

b) COMPARISON OF 1943 ORBITAL ELEMENTS

	PRESENT ANALYSIS	PORTER
EPOCH	1943 Sept. 18.0	
T, (E.T.)	1943 Sept. 22.47853	Sept. 22.574
a, (AU)	3.5596940	3.559048
e	0.610779	0.610627
Ω	143°.62188	143°.6293
i	18°.010919	18°.0114
ω	174°.38239	174°.4003

Table 14 shows the 1950 orbital elements and their 1σ uncertainty determined for the 1943-50 data fit. For prediction purposes, the uncertainty in the orbit estimate, particularly semi-major axis, must be considered with some doubt because the determined orbit does not closely represent the observations in 1923 and 1910. Nevertheless, for the sake of example, we will use these elements and uncertainty to extrapolate the orbit to the 1976 apparition.

Table 14

1950 ORBITAL ELEMENTS OF COMET D'ARREST

- 10 OBSERVATIONS IN 1943, 1950
- AVERAGE RMS FIT - 2"

ORBITAL ELEMENTS		UNCERTAINTY (1σ)
EPOCH	1950 June 11.0	
T (ET)	1950 June 6.36591	10^{-3} day
a (AU)	3.5536007	4×10^{-6}
e	0.6123469	5×10^{-6}
Ω	$143^{\circ}.60714$	3"
i	$18^{\circ}.05399$	1"
ω	$174^{\circ}.42150$	7"

3. INTERCEPT ACCURACY FOR FUTURE MISSIONS

Attention is now turned to two examples of a space probe mission to the comet - Encke in 1974 and D'Arrest in 1976. A recent survey of comet missions placed both Encke and D'Arrest on the list of selected targets for space missions during the next decade (Roberts 1965). The questions of interest in this section of the report concern the effect of comet ephemeris errors on the accuracy of spacecraft intercept, and the requirements of Earth-based comet observations in the year of launch.

3.1 Comet Encke (1974)

A summary of the mission characteristics are listed in Table 15. The assumed flight time is 110 days with a launch date of Feb. 7, 1974. The approach velocity is rather high at 35 km/sec leaving only about 40 minutes of experiment time as the spacecraft passes through the coma. As an example of payload capability, the Atlas-Centaur launch vehicle with a kick stage can deliver an 800 lb. spacecraft exclusive of shroud and adapter.

A schematic drawing of the heliocentric transfer trajectory is shown in Figure 1 along with the relative positions of the Earth and Comet Encke. Recovery is the earliest date on which the comet can be sighted from Earth, and is defined by a minimum brightness of magnitude 20 and visibility in a dark sky for a period of two hours or more. The expected recovery 160 days before launch is an important factor for this mission as will be seen shortly.

Table 15

SUMMARY OF CHARACTERISTICS FOR MISSION TO ENCKE (1974)

Mission Characteristics

Launch Date	7 Feb. 1974
Flight Time (TF)	110 days
Communications Distance (RC)	0.4 AU
Ideal Velocity (ΔV)	47,700 ft/sec
Approach Velocity (VHP)	35 km/sec
Recovery (days before launch)	160
Desired Miss Distance	1,000 km
Time Passing through Coma	40 minutes
Magnitude at Intercept	8

Launch Vehicle Payload Capability

Atlas-Centaur-Kick	800 lbs.

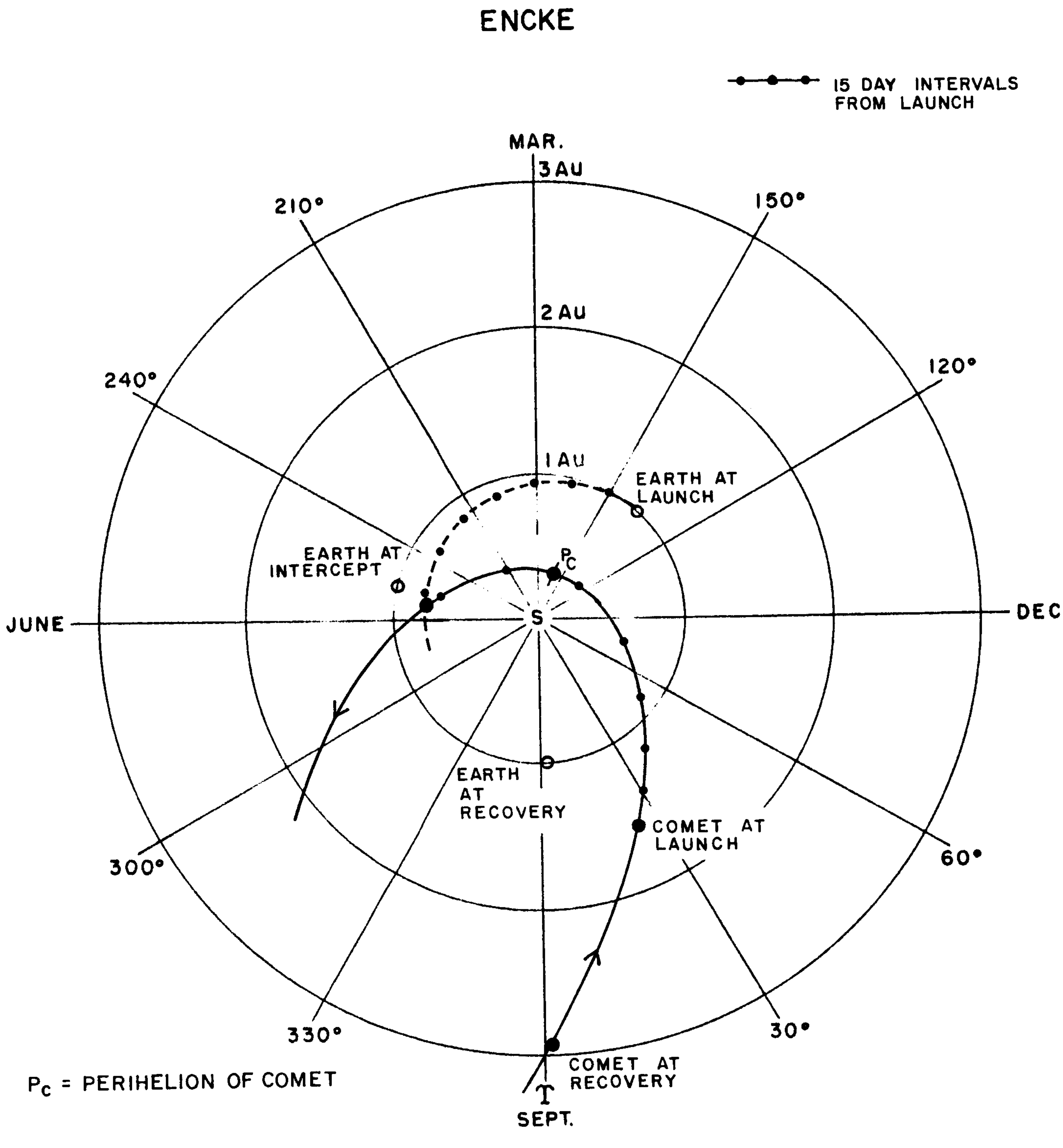


Figure 1 110 DAY TRAJECTORY TO ENCKE

Figure 2 illustrates the target plane for this mission with reference to the ecliptic plane and the positions of the Earth and Sun. Intercept occurs about 11° below the ecliptic plane. The communication distance to Earth at this time is 0.4 AU. The spacecraft approaches the comet along the S vector which is normal to the target plane (defined by the R, T vectors). T is arbitrarily defined as being parallel to the ecliptic plane. Since the approach is approximately along the direction to the Sun, the spacecraft should pass through the tail of the comet. The spacecraft miss distance lies in the T-R plane and is measured from the comet center.

Figure 3 shows the reduction in the comet's ephemeris error which is obtained by observations made in the year of launch. Plotted is the comet's tangential position error (in the direction of comet motion) at encounter as a function of successive observation times. The observation schedule begins at recovery and continues at intervals of 8 days. The assumed 2 seconds of arc observation error represents the best current practice of astrometric observation - a major part of this error is due to the position uncertainties of the background star to which the comet's photographic image is referred.

The lower curve shown in Figure 3 represents the a priori orbit information which has been extrapolated from the System II data fit; that is, from observations in 1947, 1957 and 1960. It is seen that only a few current observations are needed to reduce the position error from 60,000 km to 7,000 km.

As additional observations are made, this error remains fairly constant until March 1974 at which time the distance between Earth and Encke is rapidly reduced. This improved observation geometry causes the position error to be reduced to about 1,000 km at the time of encounter.

The upper curve in Figure 3, which assumed no a priori information, is given for the purpose of comparison. This would correspond to a worst-case situation where no confidence is given to a previous orbit determination. In this case the position error is also reduced below 10,000 km but only after 5 to 6 months of tracking.

With reference to Figure 2, the miss vector $\underline{B}$ between the spacecraft and comet is defined to lie in the R-T plane which is perpendicular to the comet approach direction $\underline{S}$ (unit vector). The relationship between $\underline{B}$ and the position uncertainty of the comet $\Delta\underline{r}_c$ is given by the vector equation

$$\underline{B} = \Delta\underline{r}_c - (\Delta\underline{r}_c \cdot \underline{S})\, \underline{S} \qquad (3)$$

It is seen that the magnitude of $\underline{B}$ is always less than or equal to the magnitude of $\Delta\underline{r}_c$. For comet orbit determination, the largest component of $\Delta\underline{r}_c$ is usually along the direction of the comet's motion, i.e., tangent to the orbit. Since the angle between the orbit tangent and the spacecraft approach direction is usually large, the second term in equation (3) is relatively small. Hence, the miss distance is determined mainly by the comet's tangential position uncertainty.

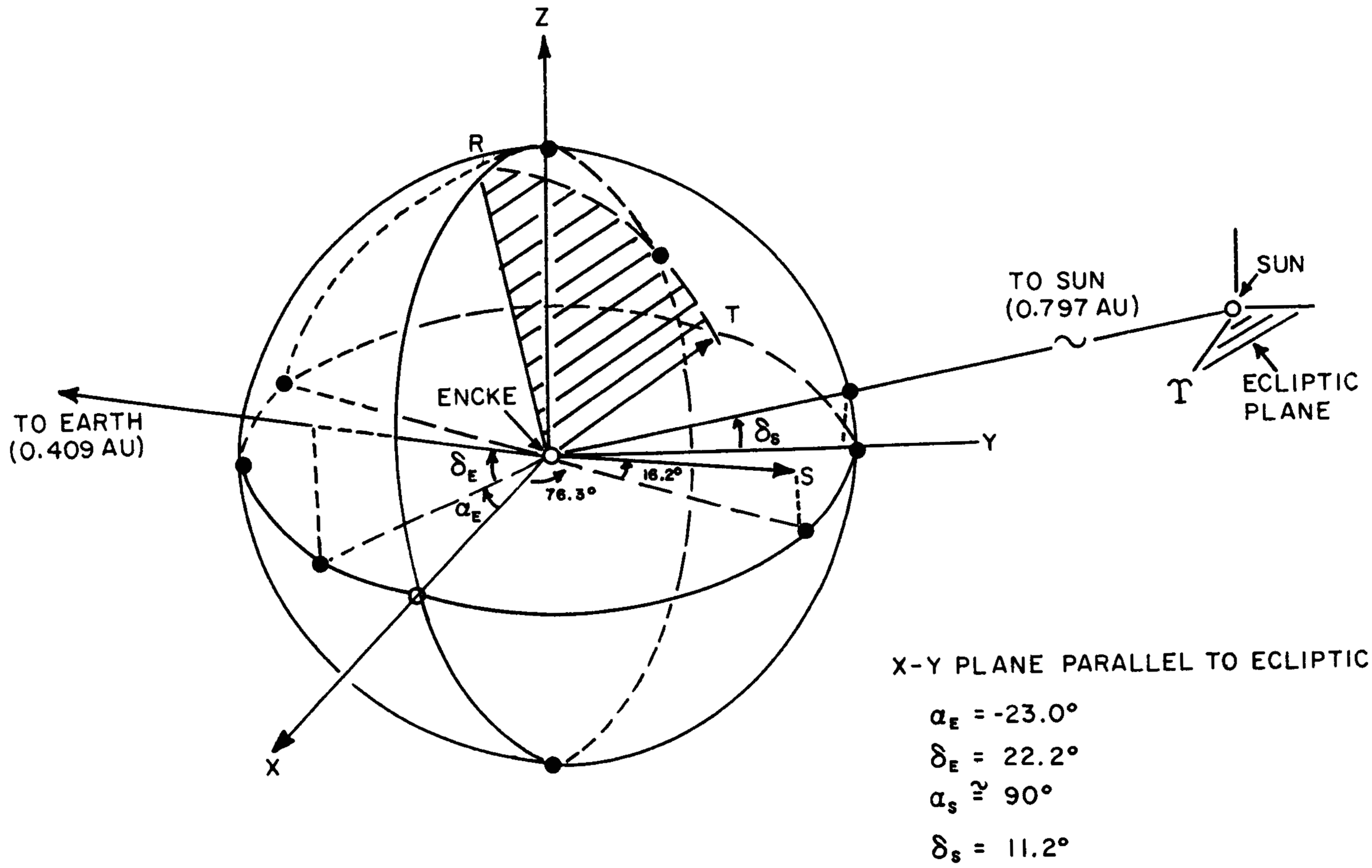


FIGURE 2. TARGET PLANE FOR MISSION TO COMET ENCKE (1974)

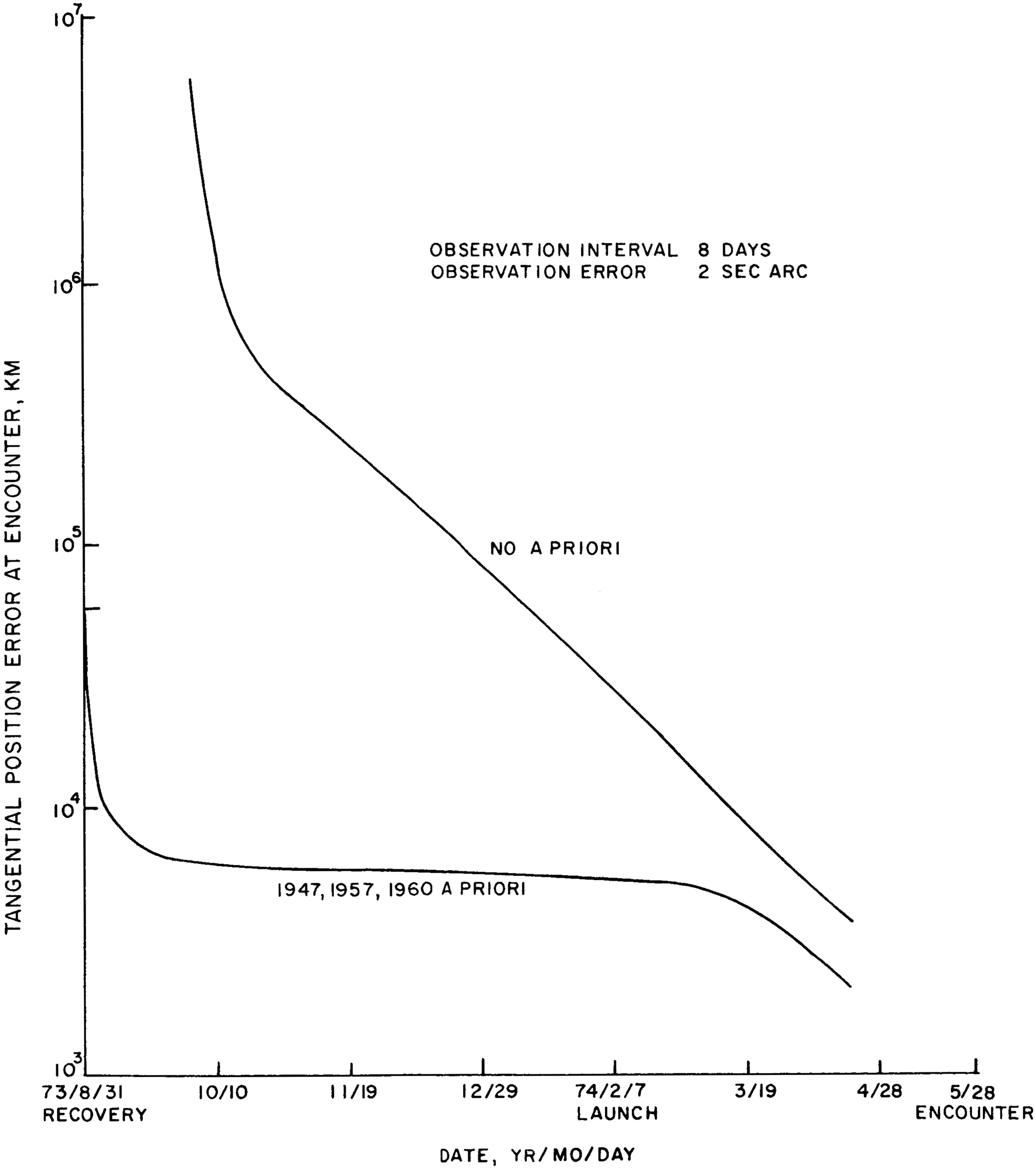


FIGURE 3. REDUCTION IN COMET'S TANGENTIAL POSITION ERROR AT ENCOUNTER WITH SUCCESSIVE OBSERVATIONS, COMET ENCKE (1974)

Figure 4 shows the spacecraft miss distance due to ephemeris error of Comet Encke. Assuming a priori information, the predicted miss distance for observations ending 1 week before launch is 7,000 km. Observations continued 4 weeks beyond the launch date only reduce the miss to 6,000 km. A miss distance below 1,000 km can be attained if the observation period is extended to within 20 days of encounter. Therefore, if small miss distances are to be obtained, a late midcourse trajectory correction is necessary. The magnitude of this correction may be approximately estimated by the expression

$$\Delta V = \frac{\text{miss distance at launch date}}{\text{time-to-go at } \Delta V \text{ execution}}$$

With a priori orbit information, a 1,000 km miss can be obtained at a ΔV cost of about 4 m/sec. Without a priori orbit information, the ΔV required is about 8 m/sec. Hence, the midcourse ΔV chargeable to the ephemeris error of Comet Encke is not very large provided the comet can be observed in the year of launch.

The distribution of the miss distance in the target plane is shown in Figure 5. This figure illustrates the extreme pencil-shaped miss ellipse that is characteristic of most comet missions. This result reflects the fact that the tangential position uncertainty of the comet is the largest contributor to the miss distance. While observations taken in the year of launch act to reduce the magnitude of the miss, they do not substantially change the orientation of the miss ellipse.

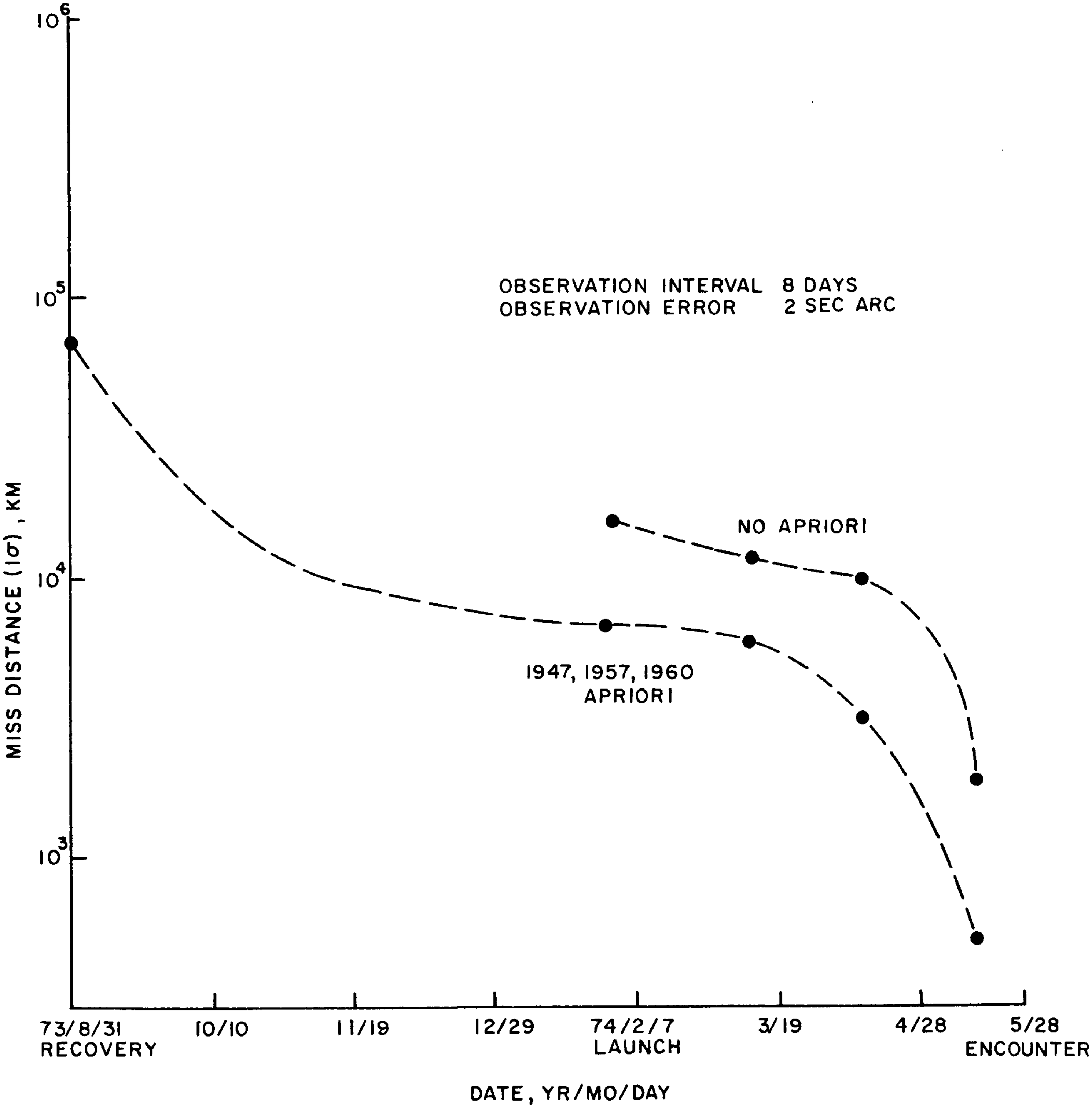
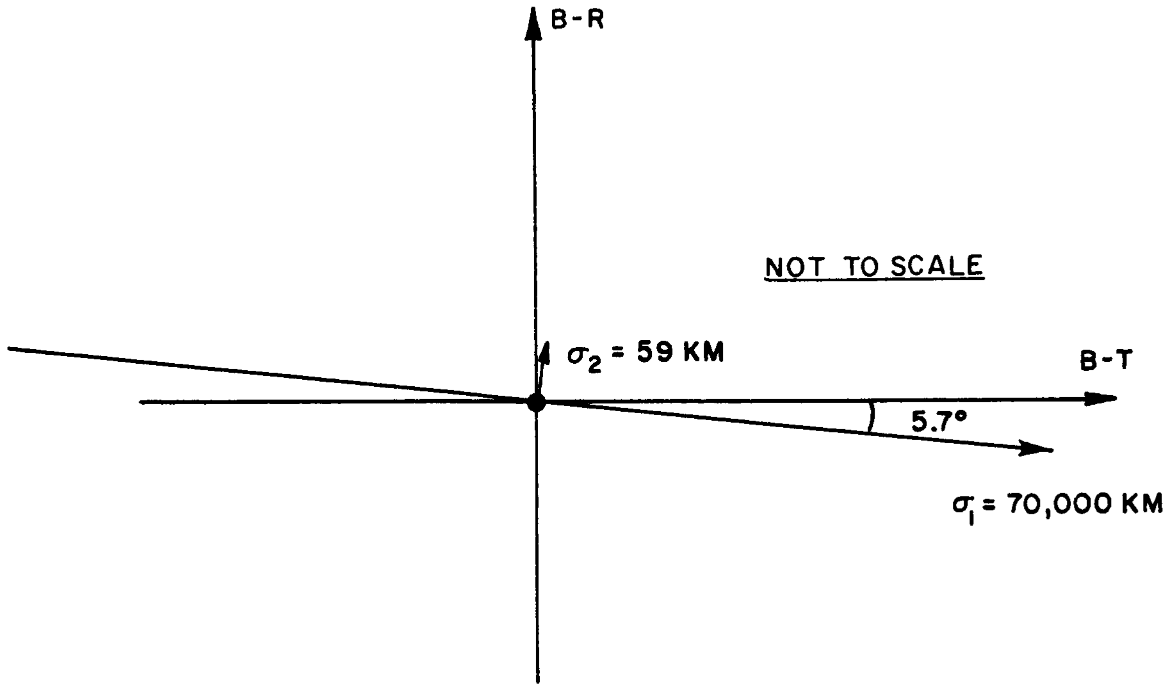
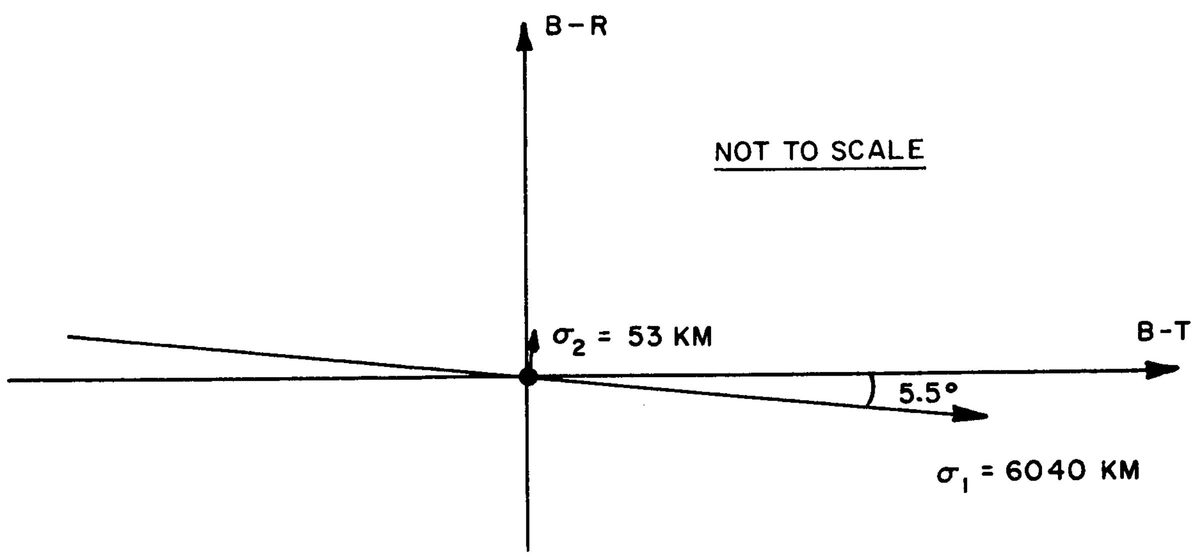


FIGURE 4. MISS DISTANCE FOR OBSERVATION TO DATE BEGINNING AT RECOVERY, COMET ENCKE (1974)



a) NO OBSERVATIONS IN YEAR OF LAUNCH, APRIORI STATISTICS MAPPED FROM BEST FIT OF 1947, 1957, 1960 OBSERVATIONS



b) 24 OBSERVATIONS FROM 1973/9/8 TO 1974/3/11 WITH 1947, 1957, 1960 APRIORI STATISTICS

FIGURE 5. ONE-SIGMA MISS ELLIPSE FOR MISSION TO COMET ENCKE (1974)

3.2 Comet D'Arrest (1976)

The characteristics of the 1976 mission to Comet D'Arrest are summarized in Table 16. The assumed flight time is 130 days with a launch date of April 21, 1976. Recovery of the comet is expected 100 days prior to launch. Figure 6 illustrates the heliocentric transfer trajectory along with the relative positions of the Earth and Comet D'Arrest. The intercept conditions are further illustrated by Figure 7, which shows that D'Arrest is about 2° below the ecliptic, and that the spacecraft approaches the comet in a direction towards the ecliptic pole and away from the Sun. The communications distance to Earth at the time of intercept is only 0.2 AU.

Reduction of the comet's tangential position uncertainty at the encounter time with successive observations in the year of launch is shown in Figure 8. As in the analysis for Comet Encke, the observation schedule begins at recovery and continues at intervals of 8 days. The initial position uncertainty resulting from the a priori orbit determination of 1943-50 is 170,000 km. Four new observations reduce this error to 10,000 km. Additional observations provide a steady improvement in the comet's ephemeris, and at the time of encounter the position uncertainty is less than 500 km. If no a priori information is assumed, a 4 month schedule of observations is required to reduce the position error to 10,000 km.

Figure 9 gives the reduction of miss distance obtained from the observation schedule in the year of launch. For observations ending 2 weeks before launch, the miss distance with and without a priori information is 4,500 km and 125,000 km, respectively. Observations continued 6 weeks beyond the launch date reduce these numbers to 1,900 km and 14,000 km. A final observation made 2 weeks before encounter brings the miss down to the desired level - 480 km in the case of apriori information and 1,050 km without such information.

The miss distribution in the target plane is illustrated in Figure 10. Again we see the tenedncy of the miss vector to define an elongated dispersion ellipse, both prior to and after including the current observations. In this case, the current observations effect a 25° rotation of the ellipse orientation towards the ecliptic plane.

Table 16

SUMMARY OF CHARACTERISTICS FOR MISSION
TO D'ARREST (1976)

Mission Characteristics

Launch Date	21 April 1976
Flight Time (TF)	130 days
Communications Distance (RC)	0.2 AU
Ideal Velocity (ΔV)	41,000 ft/sec
Approach Velocity (VHP)	13 km/sec
Recovery	100 days before launch
Desired Miss Distance	1,000 km
Time Passing through Coma	4 hours
Magnitude at Intercept	7

Launch Vehicle Payload Capability

Atlas-Agena	535 lbs.

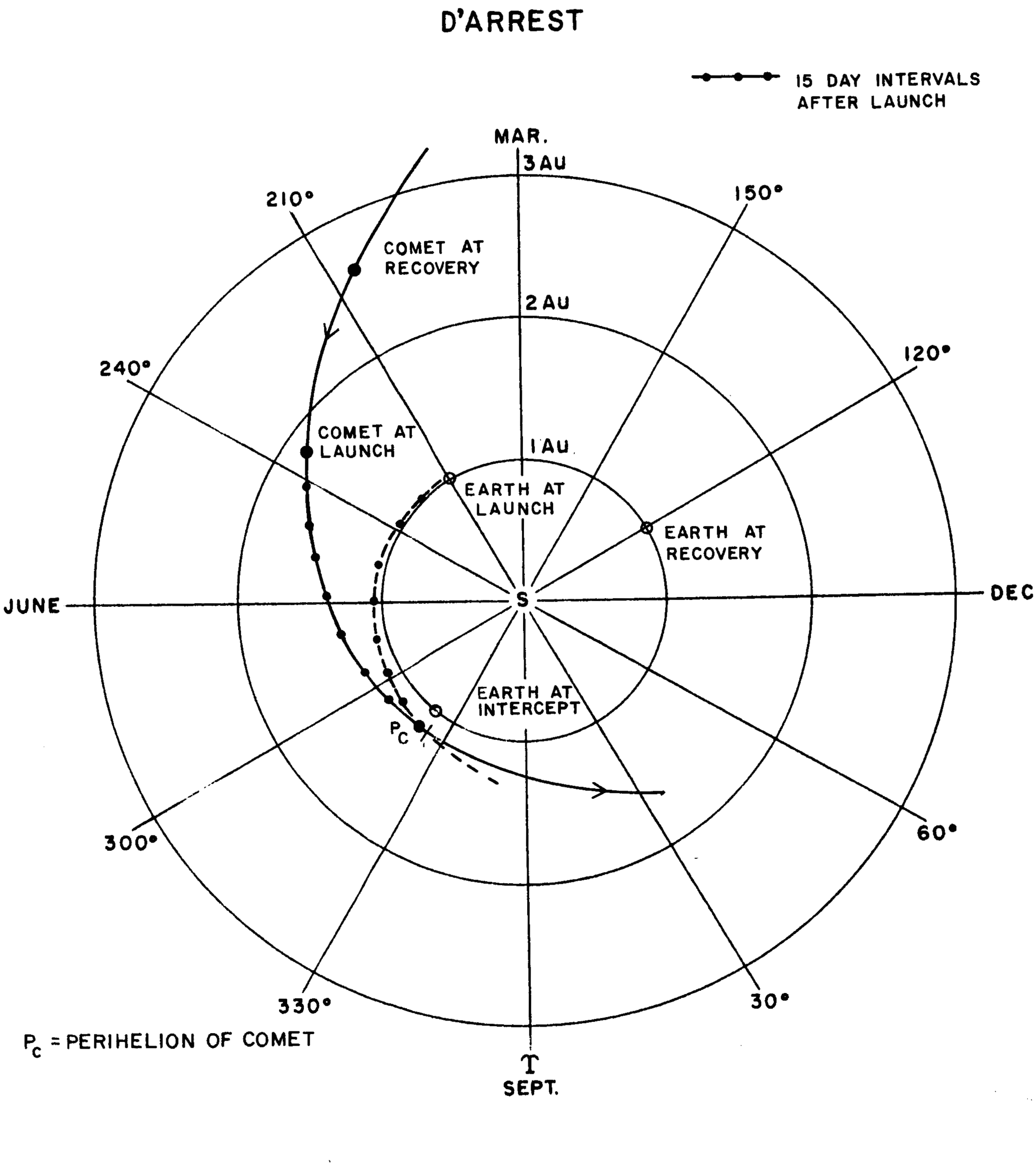


Figure 6 130 DAY TRAJECTORY TO D'ARREST

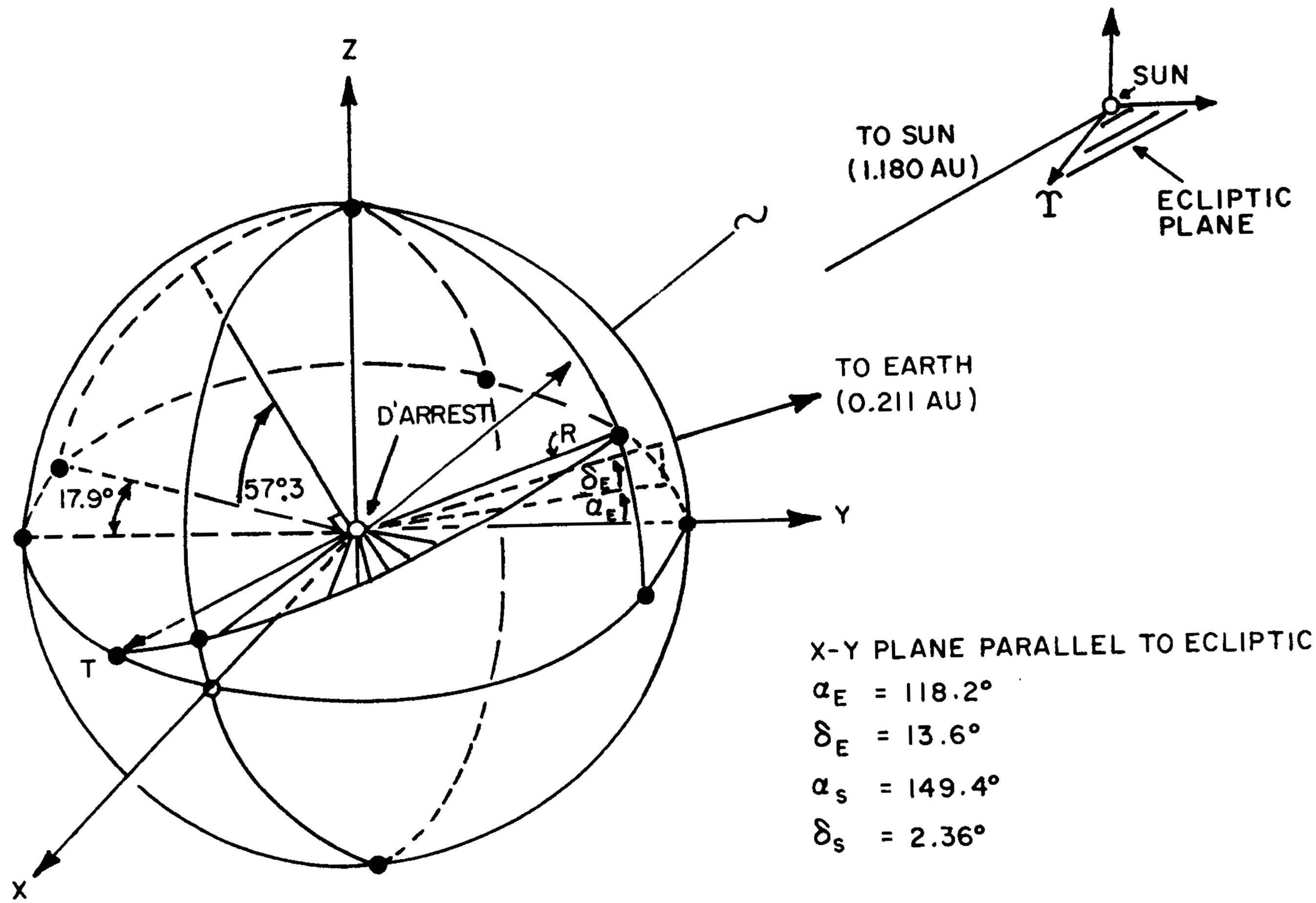


FIGURE 7. TARGET PLANE FOR MISSION TO COMET D'ARREST (1976)

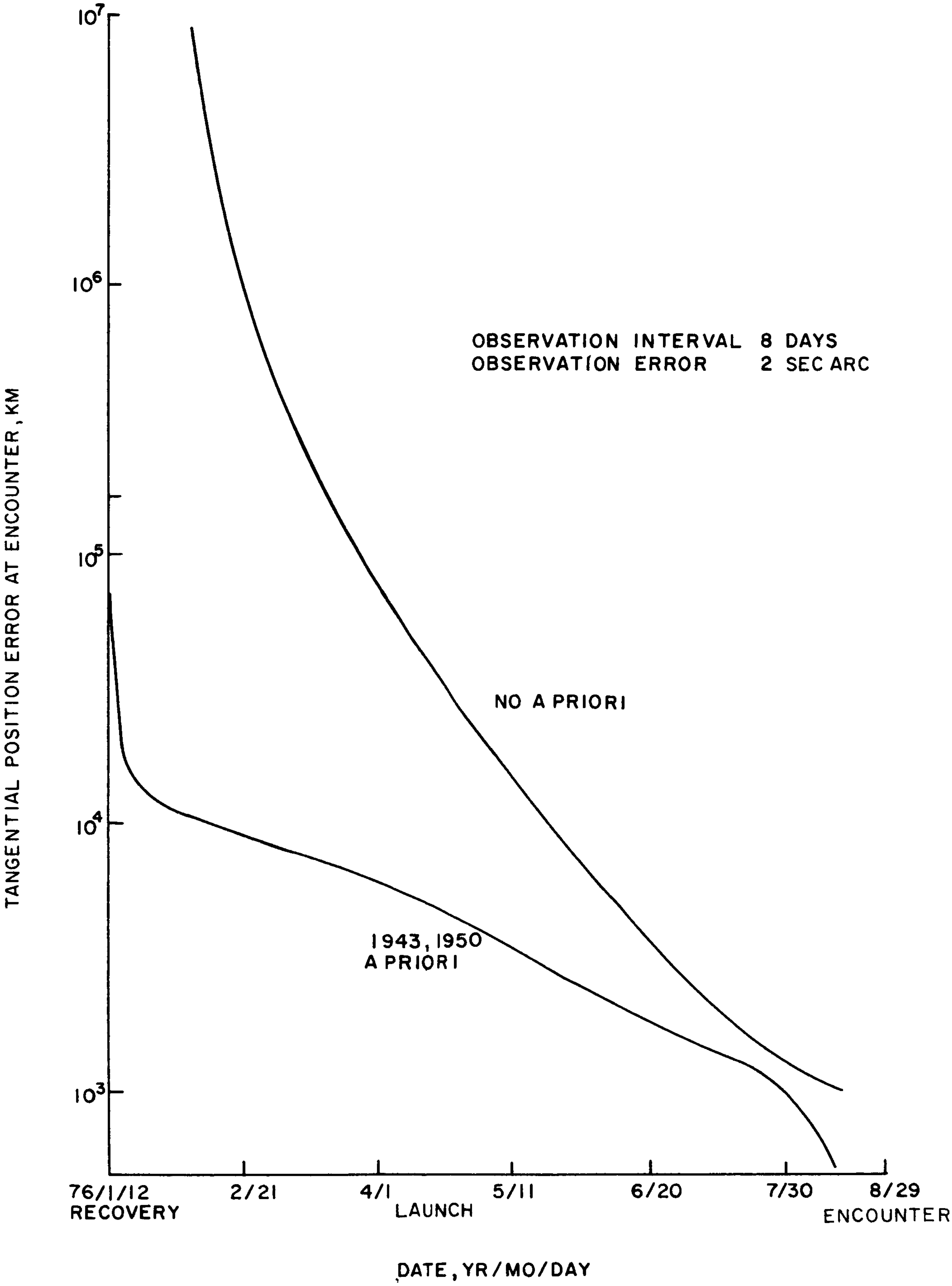


FIGURE 8. REDUCTION IN COMET'S TANGENTIAL POSITION ERROR AT ENCOUNTER WITH SUCCESSIVE OBSERVATIONS, COMET D'ARREST (1976)

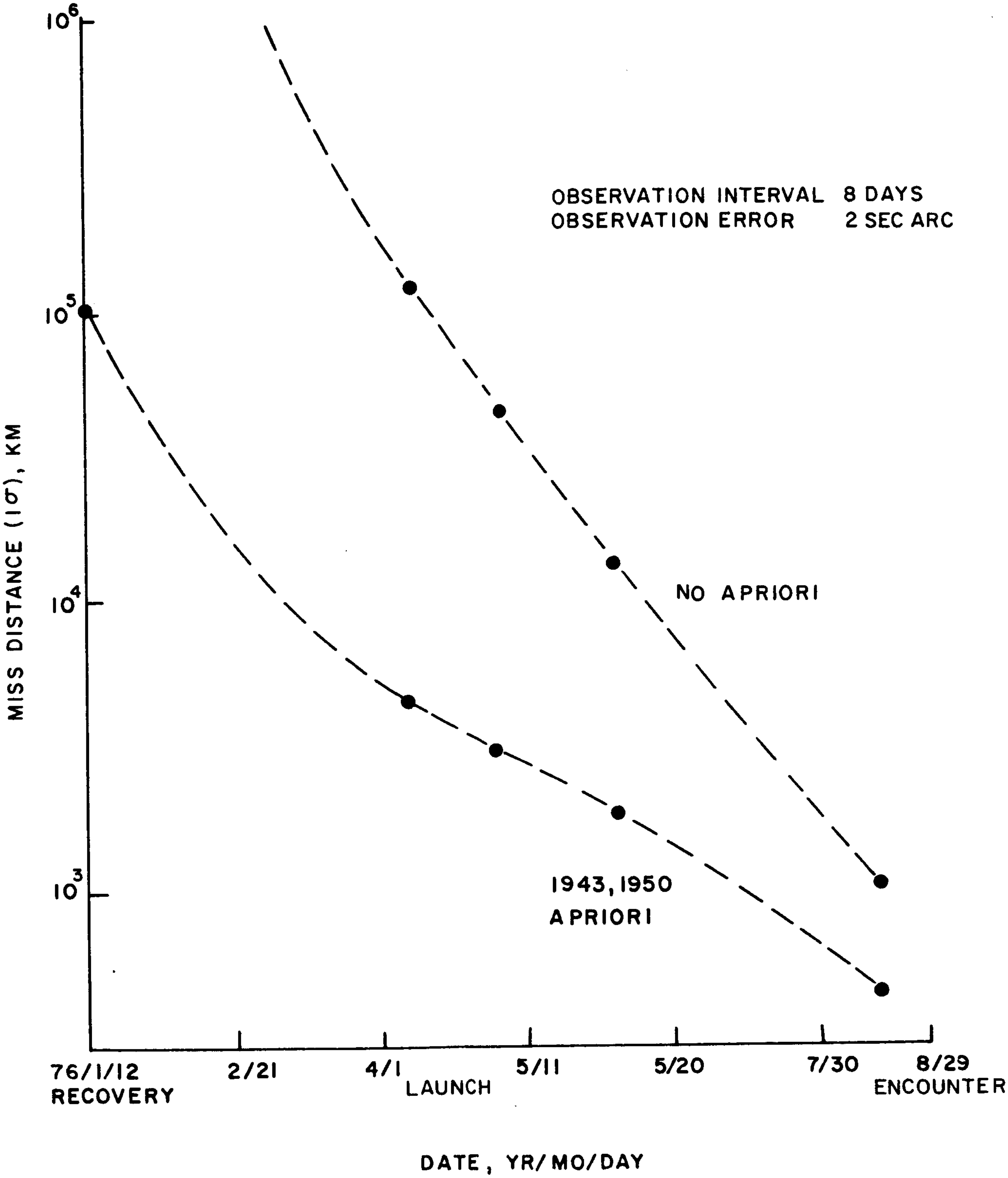
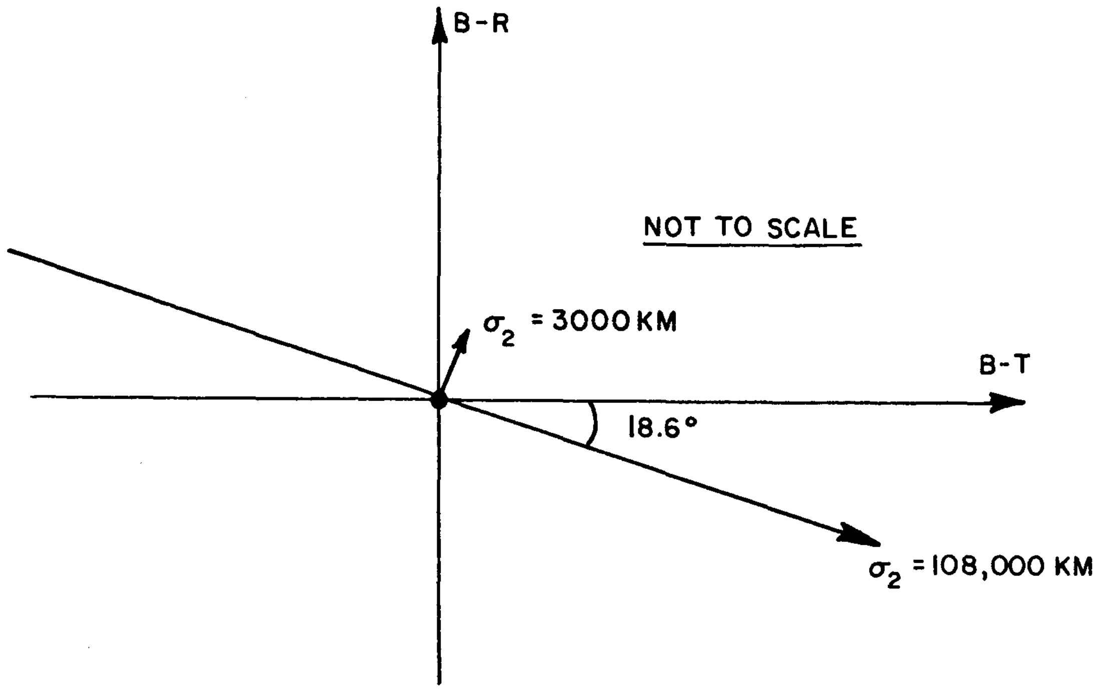
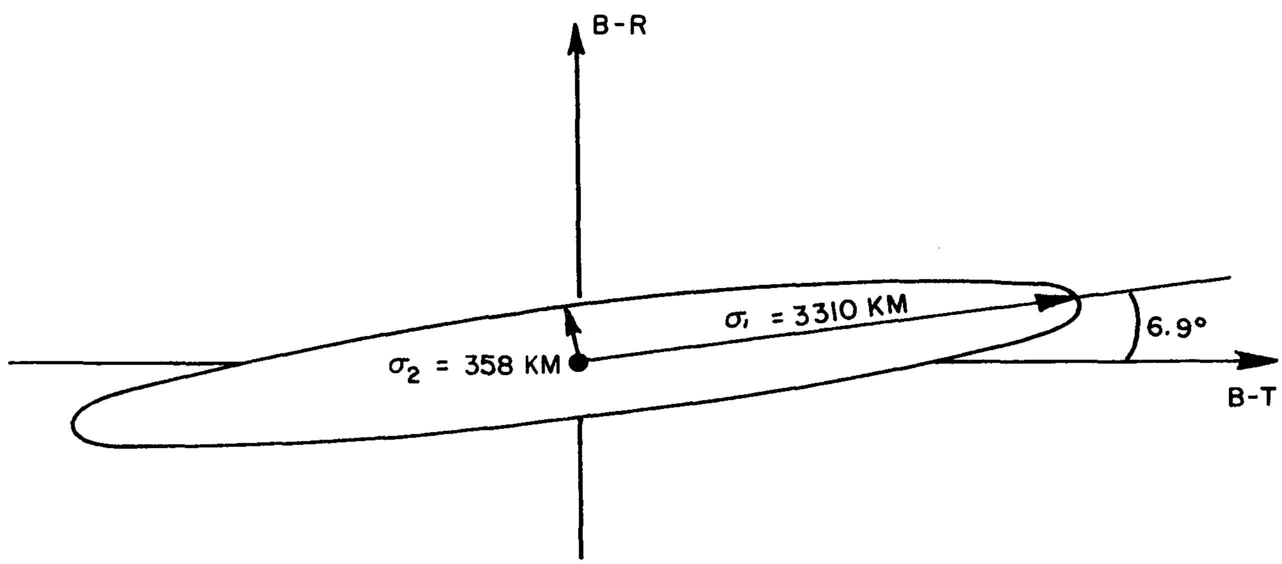


FIGURE 9. MISS DISTANCE FOR OBSERVATIONS TO DATE BEGINNING AT RECOVERY, COMET D'ARREST (1976)



a) NO OBSERVATIONS IN YEAR OF LAUNCH, A PRIORI STATISTICS MAPPED FROM BEST FIT OF 1943, 1950 OBSERVATIONS



b) 14 OBSERVATIONS FROM 1976/1/20 TO 1976/5/3 WITH 1943,1950 A PRIORI STATISTICS

FIGURE 10. ONE-SIGMA MISS ELLIPSE FOR MISSION TO COMET D'ARREST(1976)

4. CONCLUSIONS

The problems associated with the determination and prediction of cometary orbits have been explored taking Comet Encke and Comet D'Arrest as examples of short-period comets of interest for future space science missions. A fairly good representation of past observations of Comet Encke was obtained over the period 1931-1961, although it was necessary to assume a secular acceleration of motion, i.e., an acceleration due to nongravitational forces. The average value determined for this acceleration represented a decrease in orbital period of about 0.02 day/orbit. This value was in good agreement with results of earlier studies.

Less success was obtained in representing past observations of Comet D'Arrest over the period 1910-1950. In this case, the present analysis was inconclusive as to whether or not a uniform secular acceleration is in effect for D'Arrest. A tentative, but likely, identification of the source of the problem here is due to the close approach of D'Arrest to the planet Jupiter in 1920. In the first place, and perhaps most significant, is the sensitivity problem. That is, a small error in the orbit estimate will be magnified by a close approach to Jupiter so that the subsequent prediction of the motion will be degraded. This problem might be alleviated by allowing several iterations in the orbit determination process, thereby improving the validity of the linearity assumptions.

In addition to the sensitivity problem, other sources of error in the present analysis might be identified with numerical integration error buildup over many orbits, an insufficiently accurate position ephemeris for Jupiter, and an inadequate model for representing the secular acceleration if it exists.

It is recommended that further attention should be given to each of these comets, particularly Comet D'Arrest. The present analysis has obtained a very good representation (2" RMS data fit) of past observations for two recent and successive appearances of each of these comets - Encke (1957-61) and D'Arrest (1943-50). It is known that both comets were observed subsequently in 1963-4, although this data was not available for the present analysis. It is recommended, therefore, that these later observations be collected and used for an updated three-appearance determination of the comets' orbital motion. A comparison of these observations with the predicted comet ephemerides determined by the present investigation will also be useful in further verifying the accuracy of the Orbit Determination Computer Program.

The second phase of the present investigation was concerned with the effect of comet ephemeris errors on spacecraft miss distance for a 1974 mission to Encke and a 1976 mission to D'Arrest. To obtain these results the a priori information of the orbit uncertainties was extrapolated from

1961 in the case of Comet Encke and from 1950 in the case of Comet D'Arrest. It was shown that miss distances under 10,000 km cannot be achieved unless the comets are observed in the year of launch. Even then, to achieve a desirable miss distance of 1000 km or less, the observation period must extend beyond the launch date and well into the flight, thereby implying a late midcourse or terminal trajectory correction. Fortunately, in the case of both the Encke and D'Arrest missions, the comet can first be observed several months prior to launch, and the observation geometry is quite good due to the decreasing separation distance between Earth and the comet as the mission progresses. For Encke, the miss attributed to the ephemeris error can be reduced to about 6500 km by the time of launch, and then reduced further to 1000 km by executing a late midcourse maneuver 20 days before intercept for a ΔV cost of only 4 m/sec. For D'Arrest, a launch date miss of 3300 km can be reduced to 1000 km by a maneuver made 54 days before intercept at a ΔV cost of under 1 m/sec.

While the guidance accuracy and fuel requirements appear to be easily attainable for these two missions, it should be emphasized that this will not necessarily be the case for other comet missions of interest. The variable factors involved are the accuracy of previous orbit determination, the length of time between recovery or acquisition of the comet and the launch date, and the observation geometry which relates largely to the separation distance between the comet and Earth. An alternative to late Earth-based observations of the comet would be to place a comet seeker on board the spacecraft. This may be necessary anyway for those comet missions which do not have as good observation geometries as do Encke and D'Arrest. The main difficulties with an on-board comet seeker would be in the acquisition and tracking of faint, diffuse objects under the conditions of generally high approach velocities.

A final and general recommendation is that all efforts should be made to improve the ephemerides of those comets of interest for future space missions. A first requisite of precise orbit prediction is that observational data should be obtained for at least two and preferably three successive appearances just prior to the launch apparition. These observations should be linked together by a precise gravitational perturbation scheme with steps taken to eleminate or reduce systematic errors casued by observation and numerical integration. If it is found necessary (and it probably will be) to assume a non-gravitational acceleration, an adequate model of this acceleration should be sought and its best numerical estimate determined. The success of the above efforts will first, increase the probability of recovering the comet in the year of launch, and second, minimize the ΔV requirement necessitated by late comet observations, either Earth-based or on-board.

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APPENDIX A

GENERAL DESCRIPTION AND CAPABILITIES OF THE ORBIT DETERMINATION COMPUTER PROGRAM

The Orbit Determination Program for the IBM 7094 computer was developed to facilitate a moderately high precision, numerical study of the determination and prediction of cometary motion. Although the comet problem is of principal interest here, the computer program may be employed to determine the orbit of any celestial body or spacecraft given a set of angular observational data, e.g., right ascension and declination as measured from an Earth observatory.

The term "orbit determination" as used herein implies first, a definitive or most probable orbit rather than a preliminary orbit, and second, a linear differential correction to an initial orbit estimate which is sufficiently close to the true orbit. Usually, the correction is made on the basis of an over-determined set of angular observations, i.e., more than three. The observations are subject to errors of both a systematic and random nature. However, in lieu of detailed knowledge of the systematic effect, one is forced to assume that the errors are random. Techniques of statistical data processing may then be employed to compute an optimal differential correction in the sense of a most probable fit to the data. For example, the classical method of "least-squares" has often been used in past investigations. For the present program, however, we use a more recent formulation known as "sequential, minimum-variance estimation". The form of the estimator was orginally established by Kalman with the aid of linear statistical filter theory, and has since been applied extensively by others to the problem of spacecraft trajectory determination (Kalman 1960, Smith 1962, and Friedlander 1966). Functionally, sequential estimation differs from the least-squares method in that the observations are processed one at a time to yield the up-to-date best estimate of the orbit.

The Orbit Determination Program was designed to be used for the following purposes:

(1) to compute an orbit by numerical integration for a comet under the influence of the gravitational field of the Sun and perturbing planets, and other non-gravitational forces.

(2) to process actual comet observations in order to to determine the most probable estimate of the the comet's motion either in the past or future.

(3) to process simulated comet observations to be made prior to and following the launch of a comet probe for the purpose of error analysis of comet miss distance.

The following paragraphs will briefly describe the principal features of the Orbit Determination Program. This description is not, however, intended to be an operational guide to the use of the program.

A.1 Program Language, Arithmetic and Units

The program is made up of 19 functional subroutines which are written in the FORTRAN IV language, and an input load routine written in the MAP language. The input routine accepts data in the alpha-numeric form and allows arithmetic statements to be made on the data card. Single precision (8 significant figures) arithmetic is used throughout with the following exceptions: (1) the integration variables are accumulated in double precision to reduce roundoff errors, (2) time is carried in double precision to provide high resolution over many orbits, and (3) the observed and calculated angles are computed and differenced in double precision. The program operates internally in the classical units of celestial mechanics, namely, astronomical units and days.

A.2 ORBIT INTEGRATION

A.2.1 Motion Variables and Coordinate Systems

The differential equations of motion are written in the Cartesian or rectangular coordinate system referred to the mean equator and equinox of 1950.0. Position and velocity components of the body are thus the working state variables in this program. Taking the mass of the Sun as unity with all other masses measured in this unit, and the Sun as the center of coordinates, the equations of motion are expressed as

$$\ddot{x} = -k^2(1 + m)\frac{x}{r^3} - \sum_j k^2 m_j \left(\frac{x - x_j}{\rho_j^3} + \frac{x_j}{r_j^3}\right) + A_x \qquad \text{(A1)}$$

$$x \rightarrow y, z$$

Since cometary bodies have negligible mass compared to the Sun, m would be taken as zero. In the above equation k^2 is the constant of gravitation, m_j is the mass of the jth perturbing planet and x_j is its heliocentric coordinate, and A_x is the so-called secular or non-gravitational acceleration which may be acting on the comet. Also, by definition, we have the position and velocity relationships

$$
\begin{aligned}
r^2 &= x^2 + y^2 + z^2 \\
r_j^2 &= x_j^2 + y_j^2 + z_j^2 \\
\rho_j^2 &= (x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2 \qquad \text{(A2)} \\
V_x &= \dot{x} \ , \ x \rightarrow y, z
\end{aligned}
$$

The equations of motion (A1)are represented according to the Cowell form of numerical integration.

The orientation of the fundamental coordinate system is the familiar one in that the x axis is directed along the mean vernal equinox of 1950.0 and the z axis points north above the Earth's mean equator of 1950.0. Auxiliary coordinate systems used in the program are the mean ecliptic of 1950.0, and the mean equator and ecliptic of date.

In addition to the rectangular coordinates of motion, the program makes use of the standard orbital elements in the elliptical form. The orbital elements are needed for certain internal operations and input/output transformations. The principal reference frame for the elements is the mean ecliptic of 1950.0. The standard orbital element set is

(1) semi-major axis, a
(2) eccentricity, e
(3) longitude of the ascending node, Ω
(4) inclination, i
(5) argument of perihelion, ω
(6) time of perihelion, T

Auxiliary elements used are the perihelion distance (q), period (P), mean angular motion (n), mean anomoly of epoch (M_o), and longitude of perihelion ($\bar{\omega}$).

A.2.2 Secular Acceleration Model

Past investigations of cometary motion have established, with a fair degree of certainty, that forces of other than gravitational origin act to perturb the orbit. The principal effect of this perturbation is an advance or regression of the comet's position-in-orbit when compared to predicted values. In other words, the force would appear to act in the plane of motion and have a significant tangential component. To account for the possible existence of such a force, the following model is assumed

$$\underline{A} = \left(\sum_{i=o}^{N-1} p_i \ r^{-i} \right) \underline{V} \qquad \text{(A3)}$$

Here, the $\{p_i\}$ are constant coefficients to be determined by data fitting and $\underline{V}$ is the velocity vector of the comet. A maximum of 10 coefficients is allowed in the program so that one may structure the acceleration as weak or strong functions of radial distance. For example, a large value of i would, in effect, have the acceleration act predominantly in the vicinity of perihelion. While the above model was arbitrarily chosen, and does not necessarily represent any physical phenomena, it was thought to be adequate for an average fitting of a uniform change in the mean motion if such exists. It is to be noted, however, that one would have little success with this model if the acceleration were either of a random or oscillitory nature.

A2.3 Linear Perturbations

As previously mentioned, the orbit determination process is one of linear differential correction to an initial orbit estimate. The initial estimate must lie sufficiently close to the true orbit so as to validate the linearity assumption. The differential correction process necessitates having the partial derivative matrix which relates small changes in the state variables of motion from one point on the orbit to another. This matrix, frequently called the state transition matrix, is obtained as follows.

The basic state vector, consisting of the 6 components of position and velocity, is augmented by the N coefficients of the secular acceleration model. Hence, we define the state vector as

$$\underline{S} = (x, y, z, \dot{x}, \dot{y}, \dot{z}, p_o, p_i \cdot \cdot \cdot \cdot, p_{N-1}) \qquad \text{(A4)}$$

Since the $\left\{p_i\right\}$ are constant they may be represented by the differential equations $\left\{\dot{p}_i = o\right\}$. Then the equations of motion for the augmented system may be expressed in the following functional form

$$\dot{\underline{S}} = \underline{f}(\underline{S})$$

where the detailed expressions are obtained from equations (A1), (A2), and (A3). The linear perturbation equations are then given as

$$\Delta\dot{\underline{S}} = \left[\frac{\partial \underline{f}}{\partial \underline{S}}\right] \Delta\underline{S} \tag{A5}$$

$$= F(t)\Delta\underline{S}$$

with F being a square matrix having the dimension of $\underline{S}$ and implicitly varying with time. The transition matrix, Φ, also a square matrix of this dimension, is given by the differential equations and initial conditions

$$\dot{\Phi}(t,t_o) = F(t)\Phi(t,t_o) \tag{A6}$$

$$\Phi(t_o,t_o) = I \text{ (identity matrix)}$$

In general, the state transition equation relating the linear perturbations at times t_k and t_{k+1} is expressed as

$$\Delta\underline{S}(t_{k+1}) = \Phi(t_{k+1},t_k)\ \Delta\underline{S}(t_k) \tag{A7}$$

The elements of Φ are obtained by numerically integrating equation (A6) simultaneously with the nonlinear equations of motion (A1); the partial derivative elements of F being evaluated along the current estimate of the orbit. Since many elements of Φ turn out to be either zero or unity, not all of the equations need be integrated numerically. The total number of integrals obtained numerically are 6+6x6+6xN = 43+6N.

A2.4 Numerical Integration

The stepwise numerical solution of the set of differential equations is obtained by a 4th-order Runge-Kutta scheme with the option of fixed or variable step size. In the latter option, which is more efficient, the step size is controlled by a relative error comparison between the Runge-Kutta ordinate and that obtained by a lower order Simpsons rule. The step size is doubled or halved when the relation error exceeds a specified boundary (ϵ_{min}, ϵ_{max}). Values of the error boundary presently being used are (3×10^{-9}, 3×10^{-7}). Although this type of step size adjustment does not give an absolute control of truncation error, it does serve to hold the truncation error within desirable limits and also minimize the number of steps required. Round-off error is controlled by accumulating the ordinates of integration in double precision form.

The independent variable of integration, time, is measured in days from the start of integration and is carried in double precision. Time references are given by the Julian date in Ephemeris Time (E.T.) measure. Integration of an orbit may be carried out either forward or backward in time from some reference epoch. Time stops are provided in the program for the following purposes: (1) to evaluate the solution at the instants of observation, (2) to evaluate the solution at specified time intervals for output purposes, and (3) to stop the integration after a specified interval has elapsed signifying the end of a particiular case.

A.2.5 Planetary Coordinates

Provision is made to include the perturbing effects of any or all of the nine solar system planets. The planetary position coordinates needed to compute the perturbing accelerations are obtained from a set of time-varying orbital elements (Allen 1963). These elements represent the mean orbit of date in the ecliptic system, and are due to the secular part of the perturbations acting on the planets. For the purpose of the present study, it is assumed that the difference between the mean and the true osculating orbits is sufficiently small to allow the planetary perturbation accelerations to be obtained with good accuracy. The main difficulty would arise when a comet made a very close approach to one of the planets.

A.2.6 Initialization of Integration

Initial conditions of integration may be specified in either of two forms: (1) rectangular position and velocity referred to the mean equator and equinox of 1950.0, or (2) orbital elements referred to the mean ecliptic and equinox of 1950.0 or of date. In the latter option, precessional formulas are included to transform the system of date to that of 1950.0. The equatorial rectangular coordinates needed to begin the integration are then obtained by transforming through the mean obliquity of 1950.0.

A.3 ORBIT DETERMINATION

A.3.1 Observation Equations

Accurate positions of comets are determined by measurements of a photographic plate which contains the image of the comet in a background of stars whose positions are known. Right ascension (α) and declination (δ) of the comet is then found by measuring the differential coordinates and adding these to the coordinates of the comparison star(s). Published observations are usually given in topocentric rather than geocentric coordinates,and are referenced to the mean equator and equinox of some standard epoch (but not necessarily 1950.0). In order to compare the observations with a predicted geocentric ephemeris, one must first correct the observations for the effect of geocentric parallax and that part of the planetary abberation due to the comet's motion. Assuming that this correction is made, the equations relating the angular observations to the Cartesian position coordinates are

$$\begin{aligned} \rho \cos \alpha \cos \delta &= x + X \\ \rho \sin \alpha \cos \delta &= y + Y \\ \rho \sin \delta &= z + Z \end{aligned} \qquad \text{(A8)}$$

where (x,y,z) is the heliocentric equatorial position of the comet, (X, Y, Z) is the geocentric equatorial position of the Sun, and ρ is the geocentric distance of the comet. The variational expressions obtained from (A8) are, in matrix notation

$$\begin{bmatrix} \cos\delta\Delta\alpha \\ \\ \Delta\delta \end{bmatrix} = \begin{bmatrix} \dfrac{-\sin\alpha}{\rho} & \dfrac{\cos\alpha}{\rho} & 0 \\ \\ \dfrac{-\cos\alpha\sin\delta}{\rho} & \dfrac{-\sin\alpha\sin\delta}{\rho} & \dfrac{\cos\delta}{\rho} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

or

$$\Delta\underline{\theta} = H_1 \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \tag{A9}$$

Finally, since we intend to find a differential correction to the entire state vector $\underline{S}$, we may write the generalized observation equation in matrix form

$$\Delta\underline{\theta} = H\ \underline{\Delta S}$$

$$\left.\begin{aligned} H(i,j) &= H_1(i,j) \left.\right\} \begin{matrix} i = 1,2 \\ j = 1,2,3 \end{matrix} \\ H(i,j) &= 0 \left.\right\} \begin{matrix} i \neq 1,2 \\ j \neq 1,2,3 \end{matrix} \end{aligned}\right. \tag{A10}$$

A.3.2 Estimation Equations

Since the current state of motion cannot be determined precisely as a result of random observational errors, we seek a method of processing the available observational data so that the "best" estimate of the state is obtained. The meaning of "best" in this context depends upon the particular optimality criterion chosen. Quite generally, one would like to minimize some function of the error in the estimate. Specifically, one would like to choose an error function which is physically meaningful and yet leads to an easily implemented estimation procedure, e.g., linear processing of the observational data.

It has been shown by several investigators of the general estimation problem that the unrestricted optimal estimate of a linear system subject to Gaussian error statistics is of a linear form. In the event that Gaussian statistics cannot be assumed, the linear estimate is still optimal if the criterion chosen is to minimize the expected value, $E(\epsilon)$, of a quadratic error function. This is the vector equivalent of the familiar mean-square error criterion for a single variable. Given this general result, Kalman proceeded to treat the estimation problem from a dynamic filtering point of view and derived the form of the optimal linear estimation (Kalman 1960). This form is now usually referred to as "sequential, minimum-variance estimation". The estimation equations are given below without proof.

The notation used in the estimation equations is defined:

t_k, t_{k-1}	Successive time instants at which observations are given.
$\hat{\underline{S}}p(t_k)$	Predicted estimate of $\underline{S}(t_k)$ to be obtained from solution of the nonlinear equations of motion extrapolated from $\hat{\underline{S}}(t_{k-1})$.
$\hat{\underline{S}}(t_k)$	Updated estimate of $\underline{S}(t_k)$ after including the current observational data
$\Delta\underline{\theta}(t_k)$	Residual between the observed and predicted right ascension and declination, $\underline{\theta}_o(t_k)-\hat{\theta}_p(t_k)$ where $\hat{\underline{\theta}}_p$ is found from $\hat{\underline{S}}_p(t_k)$.
$N(t_k)$	Covariance matrix of the random error in the observation $\underline{\theta}_o(t_k)$, diagonal if the errors in measuring α and δ are uncorrelated.

$\mathcal{E}(t_k)$ Covariance matrix of the error in the estimate $\underline{\epsilon}(t_k) = \underline{S}(t_k) - \hat{\underline{S}}(t_k)$, i.e., $E[\underline{\epsilon}\ \underline{\epsilon}^T]$ where T is matrix transpose operator.

$W(t_k)$ Optimal filter matrix for weighting the current observation residual.

With the above definitions, the estimation procedure is given by the following recursion equations:

Updated Estimate

$$\hat{\underline{S}}(t_k) = \hat{\underline{S}}_p(t_k) + W(t_k)\ \Delta\underline{\theta}(t_k) \quad \text{(A11)}$$

Optimal Filter

$$W(t_k) = \mathcal{E}_p(t_k)H^T(t_k)\left[H^T(t_k)\ \mathcal{E}_p(t_k)H(t_k) + N(t_k)\right]^{-1} \quad \text{(A12)}$$

Predicted Covariance

$$\mathcal{E}_p(t_k) = \Phi(t_k, t_{k-1})\ \mathcal{E}(t_{k-1})\ \Phi^T(t_k, t_{k-1}) \quad \text{(A13)}$$

Updated Covariance

$$\mathcal{E}(t_k) = \mathcal{E}_p(t_k) - W(t_k)H(t_k)\ \mathcal{E}_p(t_k) \quad \text{(A14)}$$

It is to be noted that the above equations apply to any time point at which no observations are made simply by setting W=0.

A.3.3 Initialization of Estimation

As seen from the above equations, the optimal estimation procedure generates its own performance analysis. That is to say, the error covariance matrix $\mathcal{E}$ required in the computation of the filter matrix W is a measure of the accuracy obtained in determining the orbit. Specifically, the diagonal elements of $\mathcal{E}$ represent the mean-squared uncertainty in estimating the state variables of motion. The solution for $\mathcal{E}$, given by the recursion equations (A12) to (A14), is seen to depend upon (1) the initial state uncertainty $\mathcal{E}_o$, (2) the orbit variational characteristics as given by the transition matrix Φ, (3) the time sequence of observations $\{t_k\}$ and the observational geometry H at these points, and (4) the errors in the observations N.

The initial covariance $\mathcal{E}_o$ is a measure of the "a priori" accuracy of the initial orbit estimate. Thus, if one wished to assume no "a priori" information, $\mathcal{E}_o$ would be set equal to a very large value and, in effect, the orbit would be determined solely by the new observations of which at least three measures of (α, δ) are required. On the other hand, if one had a certain measure of confidence in the initial orbit estimate the value of $\mathcal{E}_o$ would be set accordingly.

Two options are available for initializing the covariance matrix: (1) $\mathcal{E}_o$ specified in rectangular position/velocity coordinates, and (2) $\mathcal{E}_o$ specified in terms of the orbital elements which is then transformed into rectangular coordinates for numerical operations. The latter option is usually employed because of its more easily visualized geometric properties.

A.3.4 Preliminary Data Processing

When determining an orbit from actual observational data, input to the main computer program is required in the form

(1) Moments of observation (in E.T.)

(2) Right ascension and declination (usually topocentric) in the mean equatorial frame of 1950.0

(3) Longitude and parallax factors of the Earth-based observatory

(4) Geocentric coordinates (X, Y, Z) of the Sun in the mean equatorial frame of 1950.0

A separate computer program is used to transform the published observational data to the required form above, and also to compute the solar coordinates at the moments of observation. Punch card output from this program then serves as input to the main program.

Conversion of the moments of observation to the Ephemeris Time measure is made when needed according to the tabulated annual corrections given in the "American Ephemeris and Nautical Almanac". Linear interpolation is justified by the slowly varying annual corrections. Published values of right ascension and declination not referred to the 1950.0 system are transformed according to the precession formulas given in the "Explanatory Supplement to the Astronomical Ephemeris and Nautical Almanac". The corrections for parallax and abberation are made in the main computer program as each observation is processed.

The geocentric coordinates of the Sun and the moments of observation are obtained by interpolation from a tape containing the coordinates at 4 day intervals over the period 1900.0 to 2000.0. The reference for this tape is Vol. 14 of the "Astronomical Papers - American Ephemeris and Nautical Almanac," which gives the coordinates to 7 significant figures. To maintain this accuracy at intermediate times, an Everett's interpolation formula is employed. Results of the interpolation were checked at various points against the table published in the 1964 edition of the "American Ephemeris and Nautical Almanac." The largest deviations were found to be ± 5 units in the seventh place.

A.4 NUMERICAL EXPERIENCE - SAMPLE CASES

A.4.1 Accuracy of Numerical Integration

A test case of the basic accuracy of numerical integration was made for Comet Encke beginning at the perihelion of 1961 and carrying the integration forward through one period without planetary perturbations. Table A-1 shows the results of the test case in terms of changes produced in the orbital elements. After integrating through one full period the semi-major axis and eccentricity show changes of only 2.6×10^{-6} and 2×10^{-7}, respectively, while changes in the orientation angles are negligible. The time of perihelion in 1974 is in error by -0.0018 days.

A.4.2 Determination of Mercury's Orbit

It was decided to test the orbit determination method for one of the solar system planets which have well established orbits and for which accurate position data is available. The planet Mercury was chosen for this example. An initial estimate of the orbit was taken from the 1967 edition of the "American Ephemeris and Nautical Almanac" for the epoch 1967 April 20.0. This initial estimate is given in terms of the mean orbital elements of date which, of course, differ from the actual osculating elements. The test problem posed was to determine the actual orbit from a series of 12 simulated observations taken at 8 day intervals beginning 1967 June 1.0. This observation schedule extends over slightly more than one period of Mercury's orbit. For this example, the right ascension and declination of Mercury are assumed to be measured in a heliocentric reference frame. The α and δ data are obtained from the position ephemeris of Mercury, mean equator and equinox of 1950.0 (Duncombe 1965).

Table A-2 lists the initial elements followed by the corrected elements determined by the observations. The principal correction is in T which, while only 0.0007 days, causes significant angular errors due to Mercury's high rate of angular motion. Table A-3 shows a comparison by residuals of the initial and corrected orbit. The residuals (observed-predicted values) of the corrected orbit are reduced to a RMS average of $0\overset{''}{.}07$ which is consistent with the 6-7th place accuracy of the position ephemeris. This example serves to demonstrate the essential correctness of the orbit determination program.

Table A-1

ACCURACY OF NUMERICAL INTEGRATION

Integration of Comet Encke (1971) for One Period with No Planetary Perturbations

Orbital Elements	Epoch 1971 Jan. 11.0	Epoch 1974 May 1.0
T	1971 Jan. 10.0	
a	2.21734280	Δa, + 0.0000026
e	0.84699264	Δe, + 0.0000002
Ω	334°23415	$\Delta\Omega$, + 0"0
i	11°97425	Δi, 0"0
ω	185°91249	$\Delta\omega$, 0"11
T+P	1974 April 30.0	ΔT, - 0.0018
P	1206.001	ΔP, + 0.0022
q	0.33926975	Δq, - 0.00000005

Table A-2

DETERMINATION OF MERCURY'S ORBIT - INITIAL AND CORRECTED ELEMENTS

Epoch 1967 April 20.0
Mean Equator and Equinox of 1950.0

	Initial Elements	Corrected Elements
a	0.38709887	0.38709924
e	0.20562814	0.20562002
Ω	$47\overset{\circ}{.}716852$	$47\overset{\circ}{.}716883$
i	$7\overset{\circ}{.}002771$	$7\overset{\circ}{.}0028054$
ω	$28\overset{\circ}{.}988200$	$28\overset{\circ}{.}989068$
T	1967 May 15.92507	1967 May 15.92575

Table A-3

DETERMINATION OF MERCURY'S ORBIT[1]- COMPARISON BY RESIDUALS

12 Observations at 8 day Intervals Beginning 1967/6/1

Date	Residuals by Initial Elements[2] $\Delta\alpha$	$\Delta\delta$	Residuals by Corrected Elements $\Delta\alpha$	$\Delta\delta$
1967/6/1	-10".50	+5".13	+0".06	-0".17
6/9	- 7.76	+4.15	-0.01	-0.13
6/17	- 6.15	+2.69	-0.01	-0.04
6/25	- 5.04	+1.35	-0.00	-0.03
7/3	- 4.08	+0.34	+0.08	-0.00
7/11	- 3.52	-0.41	+0.07	+0.03
7/19	- 3.58	-1.21	+0.13	+0.05
7/27	- 5.00	-2.64	+0.15	+0.06
8/4	- 9.13	-4.80	+0.08	-0.04
8/12	-16.98	-3.16	+0.07	-0.03
8/20	-17.95	+4.02	-0.01	-0.01
8/28	-12.70	+6.36	-0.01	+0.01
RMS Average	9.80	3.55	0.07	0.07

[1] α, δ data for this example were obtained from position ephemeris of Mercury (Duncombe 1965)

[2] Initial estimate of orbit was obtained from mean orbital elements, epoch 1967/4/20 (The American Ephemeris, 1967)

APPENDIX B

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